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Determination of material symmetries from ultrasonic velocity measurements: A genetic algorithm based blind inversion method

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Abstract

The determination of material symmetries and principle plane orientations of anisotropic plates, whose planes of symmetries are not known, are calculated using a Genetic Algorithm (GA) based blind inversion method. Ultrasonic phase velocity profiles were used as input data to the inversion. During each blind inversion, the material was initially assumed to be dependent on 21 elastic moduli (general anisotropy). A Genetic Algorithm based method was exploited to identify the ''statistically significant'' elastic moduli using the coefficient of variation (C_V) to derive a reduced set of elastic moduli. The unknown material symmetry and the principle planes (angles between the geometrical coordinates and the material symmetry coordinates) were evaluated using the method proposed by Cowin and Mehrabadi [Cowin SC, Mehrabadi MM. On the identification of material symmetry for anisotropic elastic materials. Quart J Mech Appl Math 1987;40(4):451–76], Cowin [Cowin SC. Properties of the anisotropic elasticity tensor. Quart J Mech Appl Math 1989;42(2):249–66] and Sun [Sun M. Optical recovery of elastic properties for a general anisotropic material through ultrasonic measurements. M.S. thesis, The Graduate School, University of Maine, August 2002]. This procedure was verified using simulated ultrasonic velocity data sets on materials with transversely isotropic, orthotropic, monoclinic and triclinic symmetries. Experimental validation was also performed on a unidirectional graphite-epoxy $[0]_{7s}$ material, a quasi-isotropic graphite-epoxy $[0/45/90/-45]_{7s}$ fiber reinforced composite plate, and plate cut at 45° to the fibers from a unidirectional graphite-epoxy composite plate.

Keywords: A. Structural materials; A. Carbon fibers; C. Elastic moduli; C. Statistics; D. Non-destructive testing

1. Introduction

In general, ultrasonic wave propagation velocity in an arbitrary direction with respect to the material symmetry axes is more sensitive to certain elastic moduli and less sensitive to other elastic moduli. Inversion of such velocity data results in good estimates for the more sensitive elastic moduli and relatively poor estimates for the less sensitive elastic moduli. In order to determine all the elastic moduli correctly, several propagation directions with respect to the material symmetry axes are chosen by preparing the sample and orienting it suitably such as in a goniometric set-up [4]. Goniometric immersion configurations have been used extensively to measure ultrasonic velocities at various propagation angles in different planes in materials of known symmetry. Using an appropriate inverse method, elastic moduli are then reconstructed using the Christoffel equation [5] and ultrasonic velocity data. Normally, elastic moduli are determined by minimizing the error between experimentally measured ultrasonic velocities and those calculated with an assumed set of elastic moduli [4,6,7].

It is the normal practice to carry out the inversion assuming an a priori knowledge of the material symmetry [4,6,7]. While it is frequently true (particularly in composites) that the material symmetry and principal directions

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are known a priori, there are several important examples where this is not the case. For instance, in unidirectional fiber reinforced graphite epoxy laminated composites, the assumption of transverse isotropy has been controversial due to the lack of geometrical symmetry in the plane perpendicular to the fibers. This is further complicated by the possibility of consolidation (material inhomogeniety) in the thickness direction during the manufacture of these laminated structures. Hence, some researchers have used orthotropic symmetry for modeling these structures [4]. It would therefore be desirable to be able to determine the material symmetry from measurements without a priori knowledge of material symmetry.

In this paper, a blind inversion technique is proposed for determining both the material symmetry and the principle plane directions from oblique angle ultrasonic velocity data on anisotropic plates. Using the stochastic nature of the Genetic Algorithm (GA) [8] approach, this blind inversion simultaneously calculates: (a) the elastic moduli, (b) the material symmetry class, and (c) the orientation of the principal planes. No input regarding the symmetry class of the material is required. The present method may be applied in (i) characterizing materials such as chopped fiber, 2D and 3D woven composites [9–11] in order to verify the fiber orientation, the effective material symmetry and principal directions of anisotropy that must be known for predicting the performance of these materials, (ii) characterizing the natural composites such as wood where the symmetry and principal directions are not known a priori, (iii) characterizing metals that have anisotropy due to grain orientation caused by primary/secondary manufacturing processes such as casting, rolling and heat treatment, (iv) characterizing the anisotropy of unknown crystalline materials (both natural and grown), (v) finding the changes in the stiffness tensor induced by damage such as exposure to elevated temperatures in a composites material [12], and (vi) in determining the effective anisotropic properties of functionally graded materials [13], (vii) post-fabrication checks on components designed to have specific anisotropic characteristics, and (viii) in-service quality assurance of components and structures. In this paper, a blind inversion technique is proposed for determining both the material symmetry and the principle plane directions from oblique angle ultrasonic velocity data on anisotropic plates.

2. Approach

The approach is based on the use of a simple statistical measure of the sensitivity of elastic moduli along with a powerful and efficient GA scheme. The statistical measure of the deviation ($\sigma[C_{ij}]$) of an elastic constant from its mean ($\mu[C_{ij}]$), is termed as the coefficient of variation [14] of that particular elastic constant and is defined as

$$C_{\mathrm{v}}[C_{ij}] = \frac{\sigma[C_{ij}]}{\mu[C_{ij}]} \times 100 \tag{1}$$

Table 1
Defining two second order tensors from the elastic constant matrix

(a) Voigt tensor A_{ij}		
$C_{11}+C_{12}+C_{13}$	$C_{16}+C_{26}+C_{36}$	$C_{15}+C_{25}+C_{35}$
$C_{16}+C_{26}+C_{36}$	$C_{12}+C_{22}+C_{23}$	$C_{14}+C_{24}+C_{34}$
$C_{15}+C_{25}+C_{35}$	$C_{14}+C_{24}+C_{34}$	$C_{13}+C_{23}+C_{33}$
(b) Dilatational modulus B_{ij}		
$C_{11}+C_{55}+C_{66}$	$C_{16}+C_{26}+C_{45}$	$C_{15}+C_{46}+C_{35}$
$C_{16}+C_{26}+C_{45}$	$C_{22}+C_{44}+C_{66}$	$C_{56}+C_{24}+C_{34}$
$C_{15}+C_{46}+C_{35}$	$C_{56}+C_{24}+C_{34}$	$C_{33}+C_{44}+C_{55}$

When standard deviation is large compared to the mean, the amount of variation is large. The "statistically significant" elastic moduli can be obtained if the C_{V} of all elastic moduli is less than a threshold value. It is shown in the present study that the relative insensitivity of certain elastic moduli is not arbitrary; there are weak but definite bounds on such elastic moduli. These moduli bear an unambiguous correlation to the underlying symmetry. Calculation of C_{V} for each C_{ij} element as explained in Section 3. It may be noted that a constrained search such as is employed in gradient-based methods may lead to strong bounds (or lower thresholds). However, these lower bounds are due to the search methodology rather than being a characteristic of the solution set.

The method proposed by Cowin and Mehrabadi [1], Cowin [2] and Sun [3] is used to find the symmetry and orientation of principle planes of an anisotropic material. Using the elastic stiffness matrix (C_{ij}), two second order tensors $\boldsymbol{A}_{ij}$ (the Voigt tensor) and $\boldsymbol{B}_{ij}$ (the dilatational modulus) are defined [1–3] as shown in Table 1.

Eigenvectors corresponding to each tensor $\boldsymbol{A}$ and $\boldsymbol{B}$ are calculated. Any vector is normal to a symmetry plane if and only if the vector is an eigenvector of tensors $\boldsymbol{A}$ and $\boldsymbol{B}$. From this, it can be concluded that, if three eigenvectors of $\boldsymbol{A}$ and three eigenvectors of $\boldsymbol{B}$ do not match, then the material has no symmetric plane. This would mean that it could be classified as a material with triclinic symmetry. If one eigenvector of $\boldsymbol{A}$ is equal to one eigenvector of $\boldsymbol{B}$, then the material has monoclinic symmetry. If all three eigenvectors of $\boldsymbol{A}$ and $\boldsymbol{B}$ are equal, then the material is orthotropic. At this point, the principle axis with respect to the reference plane needs to be determined. Orientation of the principle plane, R_{p}, with respect to the reference coordinate system is specified by the Euler angles and is well explained in [1–3].

3. Inversion using the Genetic Algorithm approach

Elastic moduli are related to ultrasonic velocities along different propagation directions through the Christoffel equation. From experimental bulk wave velocities, the elastic moduli are reconstructed using a Genetic Algorithm (GA) based inversion method. Genetic algorithms are robust, very efficient in finding the near global minimum (or maximum) in multidimensional search spaces and have been used extensively in obtaining elastic moduli [4,6,7,13].

The GA based inverse method of reconstruction starts with a solution set (population) of randomly guessed candidate solutions (chromosomes) of the elastic constant set which are coded as real. For triclinic, monoclinic and orthotropic symmetries, each candidate solution (chromosome) consists of 21, 13 and 9 unknowns representing, respectively, the 21, 13 and 9 elastic moduli. Variable ranges are used in generating the candidate solutions. Each candidate solution of the solution set undergoes an evolutionary process subject to some fitness criteria. Variable ranges are used in generating the candidate solutions. Each candidate solution is checked for the physical constraints [15] before it is used in the fitness calculation. The generalized physical constraints for orthotropic materials were obtained on the condition that both the stiffness and compliance matrices must be positive definite. A few of the candidate solutions with the lowest error function (or highest fitness value) are preserved, while the others are replaced with new trials. The subsequent trials are driven by the evolutionary process [6] involving crossover (jumps in solution space), creep (search space is expanded), mutation (to avoid stagnation), and elitism (to preserve good solutions).

Inversion using a GA proceeds by minimizing the error function, ERR(C), where

$$\mathrm{ERR}(C) = \sum_{i=1}^{i=2} \sum_{j=1}^{j=M} [V_{ij}^{\mathrm{c}}(\phi, \theta_r)^2 - V_{ij}^{\mathrm{m}}(\phi, \theta_r)^2]^2 \tag{2}$$

Subject to $C_{\mathrm{MIN}} < C < C_{\mathrm{MAX}}$,

where C is the elastic constant vector and C_{MIN} and C_{MAX} are the lower and upper bounds on the elastic moduli; i, j are indices specifying the mode and direction of bulk wave propagation; $V_{ij}^{\mathrm{m}}, V_{ij}^{\mathrm{c}}$ are the experimental and forward calculated phase velocities; M is the number of different directions of velocity measurements; ϕ is the azimuthal angle with respect to a material symmetry direction.

As shown in Eq. (2), ERR(C) is defined as the sum of squares of the difference in the squared velocities that are measured and calculated (using assumed values of the elastic moduli). The two summations are over the types of bulk waves that are involved and over the directions of bulk wave propagation for each type.

It may be noted that the GA parameters corresponding to the reconstruction of elastic moduli of one plate can be different from that for another plate. In order to have the same GA parameters for the reconstruction of elastic moduli from bulk wave velocities of plates of different symmetries, reconstruction has been carried out using sets of GA parameters instead of just a single set. These GA parameters have been optimized taking into account two factors, low standard deviation and low percentage of error in the reconstructed elastic moduli. The selected optimum GA parameters are given in Table 2 and are used throughout this work.

Reconstruction is illustrated for one case taking a set of GA parameters from the nine different sets of GA parameters given in Table 2. Using a set of GA parameters, 10 sets of elastic moduli are reconstructed for 10 different sets of random initial values. For the 10 sets of reconstructed elastic moduli, the mean ($\mu[C_{ij}]$), standard deviation ($\sigma[C_{ij}]$) and coefficient of variation ($C_{\mathrm{V}}[C_{ij}]$) values are calculated. Similarly, by varying the GA parameters given in Table 2, 9 different sets of $\mu[C_{ij}]$, $\sigma[C_{ij}]$ and $C_{\mathrm{V}}[C_{ij}]$ are calculated for the nine different sets of GA parameters given in Table 2. The elastic moduli and symmetry class of the material are calculated by comparing the calculated C_{V} with a threshold value. In all the simulations carried out, from the transversely isotropic case to the triclinic case as explained in Section 4, the C_{V} of all the elastic moduli was found not to exceed 20%. This upper bound on C_{V} was employed as a threshold when dealing with GA based inversions using measured data. Steps in the statistical analysis of elastic moduli are summarized in the flowchart in Fig. 1.

Table 2
(a) Set A of the GA parameters used in the inversion procedure. (b) Set B of the GA parameters used in the inversion procedure

GA Parameters used in the inversion procedure	
(a)	
Number of generations	500
Population size	50
Best population	20
Crossover rate	1
Crossover type	Single point
Creep Amount	0.01
Elites	1

	9 Different sets of GA parameters								
	1	2	3	4	5	6	7	8	9
(b)									
Mutation chance	0.15	0.15	0.15	0.20	0.20	0.20	0.25	0.25	0.25
Creeping chance	0.50	0.55	0.60	0.50	0.55	0.60	0.50	0.55	0.60

4. Case studies of inversion using simulated velocity data

Case studies are presented where the reconstruction has been carried out using simulated velocities for materials having triclinic, monoclinic, orthotropic and transversely isotropic symmetries. Simulations were carried out using elastic moduli data obtained from the literature [4,16]. From this data, phase velocities in the desired planes were calculated using the Christoffel equation. A detailed description related to using the Christoffel equation (with hypothetical material elastic moduli) for calculating simulated velocities in any propagation plane is given in [4]. These simulated velocities were then inverted using the GA approach to obtain the elastic moduli, the symmetry class of the material, and the orientation of the principal planes.

In reconstructing elastic moduli using simulated velocities, a plane oriented 45° to the 1–2 and 1–3 planes (hereafter called the 45° plane) is considered as the desired

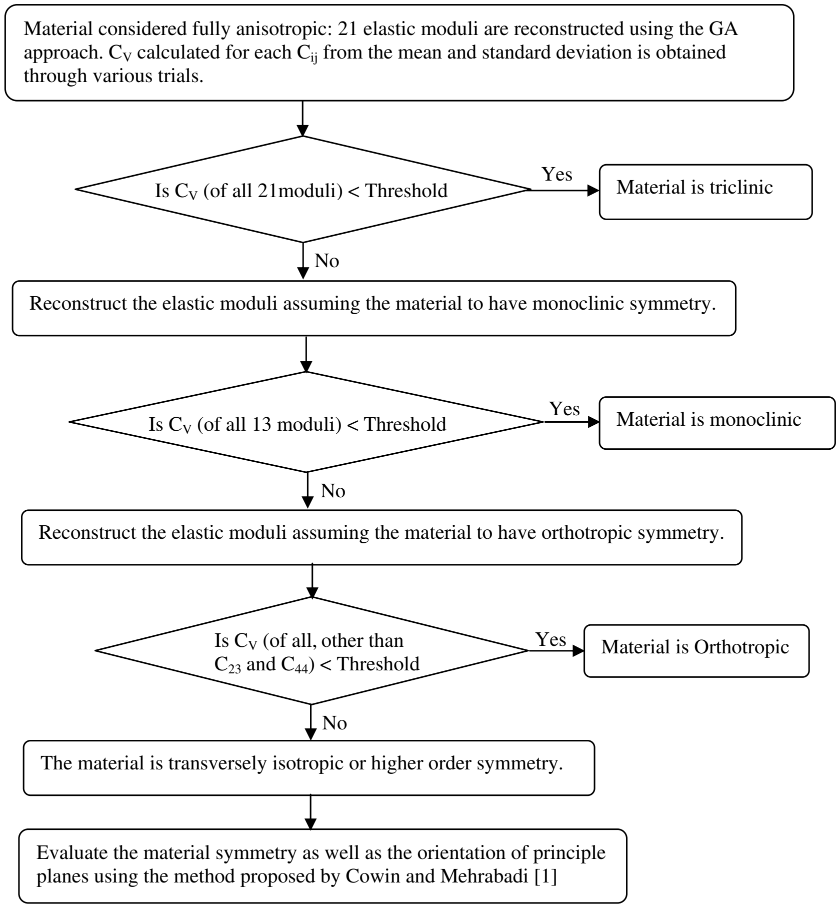

Fig. 1. General flow chart for statistical analysis of elastic moduli.

plane for triclinic, monoclinic, orthotropic and transversely isotropic assumptions. In order to show the applicability of this single plane reconstruction (where all the elastic constants are reconstructed using a single non-symmetry plane) [4], the 1–2 and 1–3 planes are used as additional desired planes for orthotropic and transversely isotropic reconstruction.

Table 3
(a) Original and (b) reconstructed elastic moduli (in GPa) of a monoclinic plate

(a)					
12.25	5.48	6.25	0.31	0.44	0.29
	20.91	23.95	13.14	1.94	1.31
		83.89	36.66	6.49	3.26
			25.51	3.57	2.14
				5.94	1.43
					4.34
(b)					
12.27	5.30	6.32	0.51	0.48	0.51
	19.73	22.59	12.59	2.03	1.90
		80.72	36.34	7.01	3.09
			25.61	3.80	2.41
				6.23	1.29
					3.92

4.1. Case study 1: Determination of monoclinic symmetry

A composite plate with monoclinic symmetry but with its symmetry axes unknown is considered. As the symmetry axes are assumed unknown, the material symmetry is regarded as triclinic, requiring 21 elastic moduli as input.

Accordingly, velocity data was simulated for wave propagation in the 45° plane using the original elastic moduli given in Table 3a [16]. Velocity data for propagation in the 45° plane was used in reconstructing the 21 elastic moduli using the GA approach. The C_V, evaluated from the mean and standard deviation of the elastic moduli, are shown in Fig. 2. From Fig. 2, it can be seen that the C_V for most of the elastic moduli are within the threshold value (20%).

As discussed in Section 1, accuracy in the reconstruction of elastic moduli depends on the relative sensitivity of velocities to the various elastic moduli. Reconstruction results in good estimates for the more sensitive elastic moduli and relatively poor estimates for the less sensitive ones. In the present case, C_{14}, C_{15} and C_{16} are relatively less sensitive. They result in poor estimates and hence C_V is above threshold. These three off-diagonal moduli, which are numerically very small, will not significantly affect the propagation velocity. The reconstructed elastic moduli

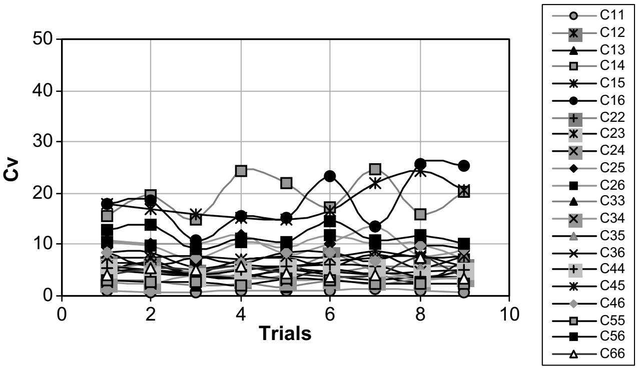

Fig. 2. Elastic moduli reconstructed using velocity data in single plane assuming medium to have monoclinic symmetry.

are shown in Table 3b and the actual elastic moduli that are used in calculating the simulated velocities are shown in Table 3a.

Table 4 shows the Voigt tensor (***A***) and dilatational tensor (***B***) constructed from the $\{C_{ij}\}$ given in Table 3b. The eigenvectors of these tensors were determined and are given in Table 5. As one eigenvector of ***A*** (3rd column in Table 5a) matches with one eigenvector of ***B*** (again the 3rd column in Table 5b), the material symmetry is taken to be monoclinic.

4.2. Case study 2: Determination of quasi-isotropic symmetry

A composite plate with quasi-isotropic symmetry but with unknown orientation of the symmetry axes is considered next. As the symmetry axes are unknown, the material is regarded as fully anisotropic requiring 21 elastic moduli as input. The simulated velocities in the 45° plane are calculated using the Christoffel equation and elastic moduli given in Table 6 as expected elastic moduli (*E*). Simulated velocity data in the 45° plane was used in reconstructing the 21 elastic moduli using the GA approach. The C_V was calculated as mentioned in Section 4.1 and are shown in Fig. 3a.

In the Case study 1 in Section 4.1, a few values of elastic moduli are above the threshold limit of C_v. In the present

Table 4
The two second order tensors, (a) Voigt tensor (A_{ij}) and, (b) dilatational tensor (B_{ij})

(a)		
23.89	5.50	9.52
5.50	47.62	49.44
9.52	49.44	109.63
(b)		
22.42	6.21	9.90
6.21	49.26	50.22
9.90	50.22	112.56

Table 5
Eigenvectors of (a) Voigt tensor (A_{ij}) and, (b) dilatational tensor (B_{ij})

(a)		
−0.07	0.99	0.10
0.88	0.02	0.48
−0.48	−0.12	0.87
(b)		
0.77	0.63	0.10
−0.59	0.64	0.48
0.24	−0.43	0.87

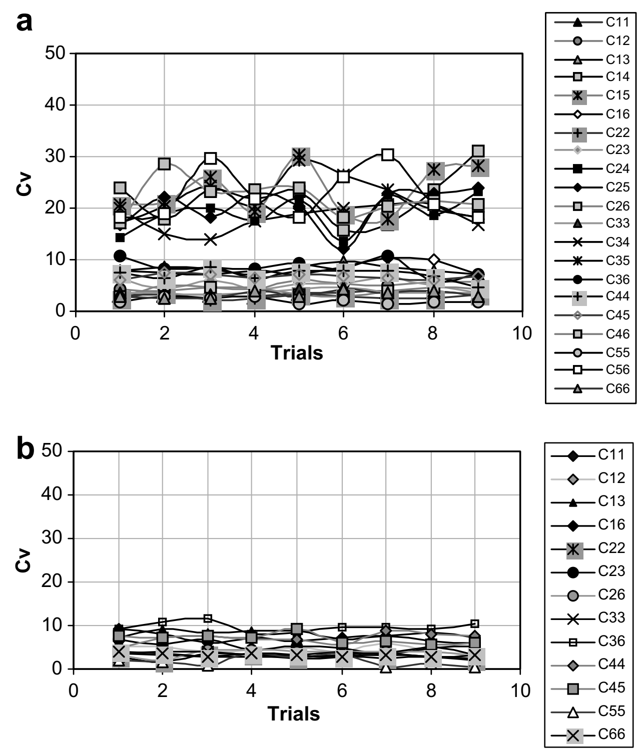

Fig. 3. Elastic moduli reconstructed using the velocity in a single plane assuming the medium to have triclinic symmetry: (a) C_v of 21anisotropic elastic moduli and (b) C_v of 13 elastic moduli after setting some elastic moduli equal to zero.

Table 6
Elastic moduli (in GPa) of a quasi-isotropic plate: Expected moduli (*E*) and reconstructed moduli (*R*) from the simulated velocities calculated using the expected moduli (*E*) given in Table 6 and assuming the medium to have triclinic symmetry

	C_{11}	C_{12}	C_{13}	C_{16}	C_{22}	C_{23}	C_{26}	C_{33}	C_{36}	C_{44}	C_{45}	C_{55}	C_{66}
E	16.06	13.85	9.49	5.46	39.39	16.84	14.74	59.10	6.37	15.53	6.51	8.01	12.29
R	15.30	12.29	9.32	4.57	40.11	16.09	15.17	60.29	5.99	15.8	6.01	8.15	11.83

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case, most of the elastic moduli are above the threshold limit. Elastic moduli which have C_v of more than 20 percent, are made equal to zero, and the reconstruction is carried out again to obtain a new set of elastic moduli and C_v as shown in Fig. 3b. The elastic moduli used to calculate simulated velocities and the reconstructed elastic moduli are shown in Table 6.

After finding the elastic moduli from the simulated velocities using statistical analysis, the symmetry class of the material as well as the orientation of the principle planes needs to be determined as explained in the Section 4.1. For the reconstructed elastic moduli given in Table 6, the eigenvectors of tensor $\boldsymbol{A}_{ij}$ and $\boldsymbol{B}_{ij}$ are given in Table 7.

To take into account any slight variation in the axes of the two tensors given in Table 7, the average, Q, of eigenvectors of tensor $\boldsymbol{A}_{ij}$ and $\boldsymbol{B}_{ij}$ is used for the further calculations. Since the eigenvectors of tensor $\boldsymbol{A}_{ij}$ and $\boldsymbol{B}_{ij}$ are equal, all three could be normals to the planes of symmetry [1]. The three eigenvectors are

$$e'_1 = -0.87e_1 + 0.49e_2 + 0e_3, \quad e'_2 = 0.49e_1 + 0.87e_2 + 0e_3, \quad e'_3 = 0e_1 + 0e_2 + e_3 \tag{3}$$

These three eigenvectors form a symmetry coordinate system, $\boldsymbol{C}$, for the material, whose reconstructed elastic moduli are given in Table 6. The components of $\boldsymbol{C}$ referred to its symmetric coordinate system are given by $\boldsymbol{C}'$. The Cartesian tensor transformation law for the fourth-rank elasticity tensor has been used to calculate $\boldsymbol{C}$.

$$C'_{ijkl} = Q_{im}Q_{jn}Q_{ko}Q_{lp}C_{mnop} \tag{4}$$

where Q_{im} is an orthogonal transformation that transforms from the coordinate system with base vectors e_1, e_2, e_3 to the symmetry coordinate system. The elastic moduli corresponding to the symmetry coordinate system are given in Table 8.

Orientation of the symmetry coordinate system (with base vectors e'_1, e'_2, e'_3 with respect to the geometry coordinate system (with base vectors e_1, e_2, e_3) is specified by the Euler angles (α, β, γ). The Euler angles are calculated by comparing elements of Q with the elements of a matrix (M) which is obtained by multiplication of coordinate transformation matrices about the e_3(or z-axis) and e_1 (or x-axis) axes by α and $\beta°$ in the clockwise direction [3].

$$[M] = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha\cos\beta & \cos\alpha\cos\beta & \sin\beta \\ \sin\alpha\sin\beta & -\cos\alpha\sin\beta & \cos\beta \end{bmatrix}$$

by comparing M with Q,

$$\alpha = \sin^{-1}[Q(1,2)] \quad \text{and} \quad \beta = \cos^{-1}\left[\frac{Q(2,2)}{\cos\alpha}\right] \tag{5}$$

Here, α and β are the Euler angles and are obtained as 29.17° and 0.07°, respectively. The expected elastic moduli (E) given in Table 6 are obtained from a 30° rotation of the elastic moduli given in Table 8 about the z-axis.

In the reconstructed elastic moduli, C_{56} shows the maximum error. The off-axis elastic moduli like the C_{56} are somewhat less sensitive to ultrasonic velocity, which makes it difficult to reconstruct accurately. However, the effect of C_{56} in finding the orientation of principle planes is very less, which can be seen from the calculated value of α is 29.17° when the actual value of α is 30°.

4.3. Case study 3: Determination of orthotropic and transversely isotropic symmetries

Elastic moduli of orthotropic and transversely isotropic used in this simulation study are given in Table 9. Simulated velocity data in the 45° plane was calculated and used in reconstructing the elastic moduli using the GA approach.

In the case of orthotropic material, the three eigenvectors of tensors $\boldsymbol{A}$ and $\boldsymbol{B}$ match, so the material is considered to have three planes of mirror symmetry and the three planes were found to be mutually orthogonal to each other. Hence the material symmetry was determined to be orthotropic and the material symmetry planes were found to match exactly with the geometric coordinate system.

In the case of transversely isotropic material, the three eigenvectors of tensors $\boldsymbol{A}$ and $\boldsymbol{B}$ match and two eigenvalues each of $\boldsymbol{A}$ and $\boldsymbol{B}$ were also found to match (eigenvalues 1 and 2), so the material was determined to be transversely isotropic and the material symmetry planes were found to match exactly with the geometric coordinate system. In both orthotropic and transversely isotropic cases, the maximum error in the reconstructed elastic moduli was found to be 0.5% and 1.35%, respectively. It was observed that error in the reconstruction of elastic moduli is relatively

Table 7
Eigenvectors of (a) Voigt tensor ($\boldsymbol{A}_{ij}$) and (b) dilatational tensor ($\boldsymbol{B}_{ij}$)

(a)		
−0.8726	0.4884	0
0.4884	0.8726	0
0	0	1
(b)		
−0.8736	0.4886	0
0.4886	0.8736	0
0	0	1

Table 8
Elastic moduli (in GPa) of a graphite-epoxy composite used to calculate simulated velocities

	C_{11}	C_{12}	C_{13}	C_{22}	C_{23}	C_{33}	C_{44}	C_{55}	C_{66}
Elastic moduli	12.43	5.81	5.81	59.10	20.52	59.10	19.29	4.26	4.26

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Table 9
Expected and reconstructed elastic moduli (in GPa) of the orthotropic and transversely isotropic plates

		C_{11}	C_{12}	C_{13}	C_{22}	C_{23}	C_{33}	C_{44}	C_{55}	C_{66}
Orthotropic	Expected	13.69	5.98	6.58	60.07	13.94	54.85	4.19	3.87	4.17
	Reconstructed	13.69	5.98	6.58	60.09	13.93	54.83	4.18	3.87	4.17
Transversely isotropic	Expected	12.43	5.39	6.24	12.43	6.24	134.36	5.00	5.00	3.52
	Reconstructed	12.43	5.37	6.30	12.43	6.16	134.36	5.01	4.99	3.53

small in the cases where the geometric coordinate system matches exactly with the material symmetry coordinate system.

5. Experimental studies

The ultrasonic velocity at different angles of incidence in a given plane was calculated from time-of-flight data measured using a single axis goniometric immersion setup; a schematic of this setup is shown in Fig. 4. The specimen is mounted between the transmitter and a back-reflector. The specimen is rotated to give different orientations for the propagation of ultrasonic waves in the material. The specimen is rotated in 1° steps. The first signal to return from the reflector is cross-correlated with the reference signal to give an accurate measurement of the time-of-flight, which, in turn, is used to calculate the ultrasonic velocity [4]. For this work, the reference was the signal without a sample (only through the water). Experiments were carried out on samples in three different propagation planes; namely, the 1–2 and 1–3 symmetric planes and a non-symmetric plane (45° to the 1–2, and 1–3 planes). A schematic of the coordinate axes of the samples was shown in Fig. 5.

From the measured time-of-flight data, the phase velocity in the material for any incident angle is calculated using Eq. (6) [4,5].

$$V_{\mathrm{p}} = \left[\left(\frac{\Delta t}{2d} \right)^2 - \frac{\Delta t/d}{V_{\mathrm{w}}} \cos\theta_{\mathrm{i}} + \left(\frac{1}{V_{\mathrm{w}}} \right)^2 \right]^{-1/2} \tag{6}$$

where V_{p} is the phase velocity (m/sec), V_{w} is the velocity in water (m/sec), d is the thickness of material (m), θ_{i} is the

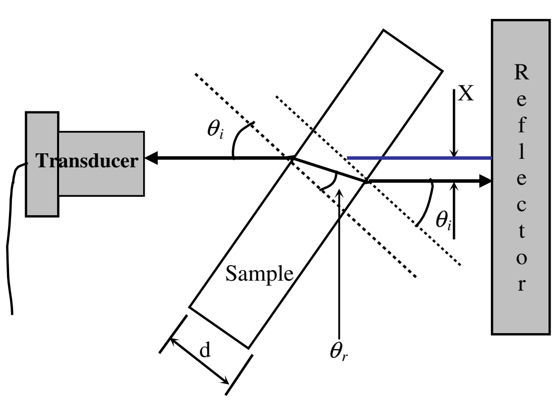

Fig. 4. Schematic of Back-Reflection Technique.

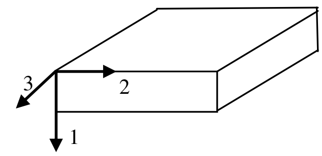

Fig. 5. Schematic of the geometric coordinate system (1–2–3) of a plane.

Incident angle (in radians), Δt is the $t_2 - t_1$, time difference between the signal with (t_2) and without (t_1) the sample present (sec).

5.1. Statistical analysis of elastic moduli using experimental velocities of 8.4 mm thick quasi-isotropic and 2.16 mm thick unidirectional graphite-epoxy composites

Three ways of reconstructing the elastic moduli have been carried out on the experimental velocities in the three different planes of the 8.4 mm quasi-isotropic (thick plate) and 2.16 mm unidirectional (thin plate) graphite-epoxy composite. Table 10 shows the mean of C_{V} for the elastic moduli. In the two reconstructions using experimental velocity data in a single non-symmetry plane, longitudinal wave velocity data has been considered from 0° to 7° and shear wave velocity data was considered from 32° to 48°.

Using the triclinic assumption, the mean of C_{V} for the 7 out of 9 orthotropic elastic moduli was found to be less than 20% but the mean of C_{V} for the other 12 elastic moduli was found to be very high for the two specimens. As the mean of C_{V} was found to be higher for anisotropic elastic moduli, triclinic symmetry was ruled out thus admitting higher symmetries. The reconstruction was carried out using velocity data in a single plane and three planes assuming the material to be orthotropic. For the two composites, the mean of C_{V} for the elastic moduli was found to be less than the threshold (20%) and hence the two composites were determined to have orthotropic symmetry.

For the calculated elastic moduli using velocity data in three planes and for an orthotropic medium, C_{V} of all the elastic moduli, except C_{23} and C_{44}, was found to be very small of the order of 4.4% (for thin plate) and 2.76% (for thick plate). The C_{V} for C_{23} and C_{44} was found to rise up to a maximum of 10% (for thin plate) and 18.68% (for thick plate). This trend reflects the well known fact even significant variations in C_{23} and C_{44} do not have any significant effect on longitudinal and shear velocities. Table 10 shows the reconstructed elastic moduli for the

Table 10
Reconstructed elastic moduli and C_V of elastic moduli (in GPa) of 8.4 mm graphite-epoxy composite (thick plate) and 2.16 mm graphite-epoxy composite (thin plate)

	Thick plate				Thin plate			
	Mean of C_V of elastic moduli			Elastic moduli	Mean of C_V of elastic moduli			Elastic moduli
	A1	O1	O3		A1	O1	O3	
C_{11}	3.50	0.00	0.00	13.69	3.36	0.00	0.00	13.48
C_{12}	5.82	4.16	1.38	5.92	9.15	9.85	2.81	7.04
C_{13}	5.29	3.39	2.76	6.38	10.33	5.47	1.88	6.95
C_{14}	27.49				29.00			
C_{15}	32.86				27.65			
C_{16}	35.19				23.03			
C_{22}	4.18	3.41	1.72	58.70	5.18	5.70	2.67	14.93
C_{23}	20.23	15.09	18.68	8.16	10.56	10.91	4.76	12.53
C_{24}	26.31				26.56			
C_{25}	30.36				27.89			
C_{26}	30.94				22.61			
C_{33}	4.44	3.30	2.67	54.59	8.42	2.52	1.35	137.60
C_{34}	27.07				22.81			
C_{35}	32.16				20.97			
C_{36}	34.10				22.07			
C_{44}	20.54	11.83	11.56	6.56	9.47	10.87	9.69	7.85
C_{45}	32.85				22.79			
C_{46}	36.27				28.35			
C_{55}	7.13	6.27	0.33	3.85	8.66	4.05	0.41	6.07
C_{56}	38.52				28.03			
C_{66}	7.08	6.39	0.19	4.18	9.26	7.34	4.38	3.82

A1 – Reconstructing the elastic moduli using single plane velocity data assuming triclinic symmetry.
O1 – Reconstructing the elastic moduli using single plane velocity data assuming orthotropic symmetry.
O3 – Reconstructing the elastic moduli using three plane velocity data assuming orthotropic symmetry.

quasi-isotropic and the unidirectional graphite-epoxy composite plates, respectively. The three eigenvectors of the tensors ***A*** and ***B*** match so there are three principle planes. Hence, the material is orthotropic with the principle planes exactly matching with the reference coordinate system.

5.2. Statistical analysis of elastic moduli using experimental velocities of the 2.16 mm thick graphite-epoxy composite plate cut at 45° to the fibers in this unidirectional composite

In order to show the applicability of the present method on samples where the orientation of principle coordinates do not coincide with the geometric coordinate system, a rectangular plate has been cut at 45° to the fiber direction from the 2.16 mm thick graphite-epoxy composite specimen. Reconstruction of the elastic moduli has been carried out using experimental velocities measured in the 45° plane of this plate. In the reconstruction, longitudinal wave velocity data has been considered from 0° to 6° and shear wave velocity data from 13° to 33°.

Table 11 shows that with the triclinic assumption, the mean of C_V for 11 out of 21 elastic moduli were found to be less than 20% while the mean of C_V for the other 10 elastic moduli is higher than 20%. As the mean of C_V is above threshold for most of the elastic moduli, the material is not triclinic. With the monoclinic assumption, reconstruction is carried out using velocity data in the 45° plane. It is observed from the Table 11 that the mean of C_V for all the elastic moduli is less than the threshold (20%). Hence, this material has monoclinic symmetry in the geometric coordinate system and the elastic moduli are shown in Table 11.

Table 11
C_V of elastic moduli (in GPa) of the plate cut from the 2.16 mm thick graphite-epoxy composite plate

	Mean of C_V of elastic moduli		Reconstructed elastic moduli (GPa)
	Triclinic assumption	Monoclinic assumption	
C_{11}	0.96	0.65	13.41
C_{12}	8.24	11.16	5.91
C_{13}	9.71	10.99	5.97
C_{14}	25.91	17.62	0.51
C_{15}	22.50		
C_{16}	22.60		
C_{22}	6.72	5.93	46.13
C_{23}	5.57	4.61	33.16
C_{24}	5.42	4.39	30.30
C_{25}	26.46		
C_{26}	24.92		
C_{33}	6.73	6.24	46.70
C_{34}	5.67	4.93	30.38
C_{35}	24.10		
C_{36}	25.11		
C_{44}	4.45	3.38	32.03
C_{45}	26.17		
C_{46}	25.84		
C_{55}	7.89	6.22	5.08
C_{56}	22.53	14.27	0.91
C_{66}	7.71	6.76	4.97

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Table 12
Eigenvectors of (a) Voigt tensor ($\boldsymbol{A}_{ij}$) (b) dilatational tensor ($\boldsymbol{B}_{ij}$) and (c) Q matrix

(a)		
0	1	0
−0.7089	0	0.7053
0.7053	0	0.7089
(b)		
0	1	0
−0.7091	0	0.7052
0.7052	0	0.7091
(c)		
0	1	0
−0.7090	0	0.7052
0.7052	0	0.7090

The three eigenvectors of the tensors $\boldsymbol{A}$ and $\boldsymbol{B}$ calculated for the elastic moduli given in Table 11 are shown in Table 12. The eigenvectors of $\boldsymbol{A}$ and $\boldsymbol{B}$ were found to match with each other hence the material is orthotropic. The average of the eigenvectors is denoted as Q matrix, which relates the geometric coordinate system with the material symmetry plane. The Q matrix along with the Cartesian tensor transformation law given in Eq. (4) is used in transforming the elastic moduli corresponding to the geometric coordinate system to the elastic moduli corresponding to the material symmetry plane.

The Q matrix is used in calculating the Euler's angles by comparing elements of Q with the elements of a matrix (M) as explained in Section 4.2. The angles are calculated as follows:

$$\alpha = \cos^{-1}[Q(1,1)]$$

$$\beta = \cos^{-1}[-Q(2,1)/\sin(\alpha)]$$

The Euler angles α andβ are obtained as 90° and 44.85°, respectively. However, the error in determining the orientation of principle planes is much smaller which can be observed by comparing the calculated value of β, 44.85°, with the actual value of 45°.

6. Discussion

In the first simulation study, many of the reconstructed moduli are close to the elastic moduli used to calculate simulated velocities and a few of the reconstructed moduli have high error due to the low magnitude of the moduli. Even though the reconstructed moduli represent triclinic symmetry, calculation using the Cowin and Mehrabadi method indicates it is a monoclinic material. In the second simulation study, error in most of the reconstructed moduli is small. C_{16} has 16% error because C_{16} is relatively insensitive to the ultrasonic velocity. Error in the reconstructed principle plane angles is small.

In all three simulation studies, the ranges of elastic constants are considered as 50% variation on both sides. Most of the commercially used composite materials have orthotropic or higher symmetry. In a case where prior knowledge of the range of elastic moduli is unknown but where ultrasonic velocity data is available in three planes (1–2 plane, 1–3 plane and a plane at 45° to 1–2 and 1–3 planes), reconstruction with reasonably accuracy can be carried out using a large search space for each variable. The material is assumed to be orthotropic while keeping the range of all elastic moduli as high as 1–1000 GPa. Reconstructed elastic moduli, constraining all the elastic moduli to ranges of 1–200 GPa, 1–400 GPa and 1–1000 GPa are as shown in Table 13. The reconstructed elastic moduli are approximately the same even when the range of elastic moduli is increased. The simulated velocities used in the reconstruction are calculated by using the theoretical elastic moduli given in Table 13. The theoretical elastic moduli are calculated using the elastic moduli provided by the manufacturer for a single ply and then rotating the layers to the different required angles and averaging the moduli depending on the lay-up of the plate.

When velocity data is available on a single non-symmetry plane, the reconstruction can be carried out in two steps. The first carrying out the search with large bounds on the elastic moduli and the second carrying out a search by reducing the search space using results obtained from the first step. The reconstructed elastic moduli obtained after these two steps are given in Table 13.

Table 13
Reconstructed elastic moduli (in GPa) with large bounds on the elastic moduli

	Theoretical	Reconstructed elastic moduli using velocity data in three planes with large bounds on elastic moduli			Reconstructed elastic moduli using velocity data in a single non-symmetry plane with different ranges of elastic moduli	
		1–200 (GPa)	1–400 (GPa)	1–1000 (GPa)	Elastic moduli from Step1	Elastic moduli from Step2
C_{11}	13.69	13.69	13.69	13.69	13.69	13.69
C_{12}	5.98	5.98	5.99	5.96	4.54	5.37
C_{13}	6.58	6.58	6.58	6.54	7.20	6.69
C_{22}	60.07	60.09	60.07	59.84	54.83	52.35
C_{23}	13.94	13.93	13.92	13.56	12.50	8.81
C_{33}	54.85	54.83	54.85	54.32	56.77	51.85
C_{44}	4.19	4.18	4.16	4.03	5.88	9.53
C_{55}	3.87	3.87	3.87	3.91	4.22	4.16
C_{66}	4.17	4.17	4.17	4.20	3.97	4.07

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Experimental validation was performed on velocity data obtained from a unidirectional graphite-epoxy composite plate, a quasi-isotropic graphite-epoxy fiber reinforced composite plate, and a plate cut from a unidirectional graphite-epoxy composite at 45° to the fibers. In the case of both unidirectional and quasi-isotropic graphite-epoxy composite plates, the material symmetry planes were found to match exactly with the geometric coordinate system of respective plates. In the case of the unidirectional graphite-epoxy composite plate cut at 45° to the fibers, the material symmetry planes were reconstructed and found to have a cut angle of 44.85° with respect to the geometric coordinate system.

In the case of reconstructing the elastic moduli using experimental velocities that contain "noise", it may be better to use single plane velocity data as opposed to using three plane velocity data where the experimental errors in three planes may effectively magnify the error in the reconstructed elastic moduli.

7. Conclusions

It was demonstrated that, using a Genetic Algorithm (GA) based blind inversion method, material symmetries and orientation of the principle planes of anisotropic plates may be obtained by sectioning them in any arbitrary orientation. The material symmetry may also be of any standard type (isotropic, transversely isotropic, orthotropic, monoclinic, or triclinic) and the technique reported here has the capability to recognize the symmetry type without any prior information provided to the algorithm. This GA technique initially determines the elastic moduli corresponding to the geometric coordinate system and subsequently provides the elastic moduli corresponding to the material symmetry coordinate system as well.

For the most general anisotropy case considered here (of triclinic material), a small data set (from ultrasonic velocity data in a single plane) gave errors in the reconstruction of elastic moduli of the order of less than 10% for most of the elastic moduli (except for some elastic moduli whose magnitudes are under 1 GPa). As shown in Section 4.2, error in determining the orientation of principle planes by the present method is less than 1%. The error in reconstruction of elastic moduli is relatively small in the cases where the geometric coordinates match exactly with the material symmetry coordinate system (as shown in Sections 4.3). In these cases, the maximum error was found to be less than 0.5% in the case of orthotropic and 1.35 % in the case of transversely isotropic material.

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