Available online at www.sciencedirect.com
ScienceDirect
Composites: Part B 38 (2007) 216–227
composites Part B: engineering
www.elsevier.com/locate/compositesb

Genetic algorithm reconstruction of orthotropic composite plate elastic constants from a single non-symmetric plane ultrasonic velocity data

J. Vishnuvardhan [b], C.V. Krishnamurthy [a], Krishnan Balasubramaniam [b,*]

[a] Center for Nondestructive Evaluation, Indian Institute of Technology, Chennai 600 036, India
[b] Department of Mechanical Engineering, Indian Institute of Technology, Chennai 600 036, India

Received 12 March 2006; accepted 9 June 2006
Available online 28 September 2006

Abstract

This paper reports a Genetic Algorithm (GA) based reconstruction procedure to determine the elastic constants of an orthotropic plate from ultrasonic velocity data. Phase velocity measurements are carried out using ultrasonic back-reflection technique on laminated unidirectional graphite–epoxy $(0)_{16}$ and quasi-isotropic graphite–epoxy $(+45, -45, 0, 90)_{7s}$ fiber reinforced composite plates. A forward model to generate the slowness curves from elastic constants has been used to verify the quality of the reconstruction. The sensitivity of the chosen GA parameters is studied. As expected, out of 9 orthotropic elastic constants to be determined, the C_{23} and C_{44} found to be insensitive. The GA based reconstruction using data obtained from multiple planes were evaluated and it is shown that the single plane reconstruction at a non-symmetric plane was sufficient for the computation of the seven elastic constants.

Keywords: A. Layered structures; B. Mechanical properties; D. Non-destructive testing; Genetic algorithm

1. Introduction

Evaluation of the mechanical behavior of composite materials [1] under severe loading conditions requires the knowledge of elastic constants of the material under consideration. Conventional techniques like tensile and compressive tests are destructive in nature and provide only a few elastic constants and are difficult to perform on thin plate like structures. Ultrasonic techniques are advantageous in these aspects over the conventional techniques and are uniquely qualified for non-destructive measurement of several of the elastic constants of such materials.

Elastic constants are related to ultrasonic velocities along different propagation directions through the Christoffel equation [2–4]. Thus it is possible to evaluate elastic wave velocities in any direction given the elastic constants. Similarly, when elastic wave velocities are measured from experiments, in principle, it is possible to deduce the elastic constants.

In practice, with normal incidence velocity data, only a few elastic constants can be determined through a simple inverse procedure. For instance C_{11} can be calculated directly from the ultrasonic velocity at normal incidence. But to find the other elastic constants, it becomes necessary to use the oblique incidence velocity data, coupled with a suitable inversion technique. The inversion problem (discussed in Appendix A) contains trigonometric terms and hence it is non-linear and non-unique in nature. The use of over determined data sets (data obtained from multiple angles of incidence) cancel out the random error in the experimental data.

The determination of elastic constants from ultrasonic bulk wave velocity data along planes of symmetry of the plate has been studied by Chu and Rokhlin [5]. They have shown that quasi-longitudinal and quasi shear velocity

* Corresponding author. Fax: +91 44 2257 0545.
E-mail address: balas@iitm.ac.in (K. Balasubramaniam).

1359-8368/$ - see front matter
doi:10.1016/j.compositesb.2006.06.006

data in two accessible planes of symmetry can be used to obtain a complete set of five independent elastic constants for a transversely isotropic material and seven of the nine independent elastic constants for an orthotropic material. They have used a non-linear least square optimization method to reconstruct the elastic constants of the plate. However, this inversion procedure involves an initial guess of the elastic constants being computed, and often these guessed values must be somewhat close to the actual value for the inversion to converge.

Ditri [6] empirically showed that for anisotropic media, of the 21 elastic constants, 15 elastic constants can be uniquely determined from velocity data measured in one plane and 20 elastic constants can be uniquely determined from data measured in two planes. Chu, Degtyar, and Rokhlin [7], reported the analysis of the reconstruction of elastic constants from ultrasonic velocity measurements in non-symmetry planes of unidirectional composite materials. They found that, when all nine elastic constants of orthotropic material were reconstructed from velocity data in non-symmetry planes, the inversion is highly dependent on the initial guesses and susceptible to random data scatter. Once, the seven elastic constants are found from velocity data in planes of symmetry, the remaining two can be found from non-symmetry-plane data independent of initial guesses and scatter levels.

Balasubramaniam and Whitney [8] have calculated elastic constants of thick unidirectional and cross-ply glass-epoxy composite plates. The authors measured group velocity as a function of energy propagation angle, from which they computed phase velocity as a function of phase angle. Phase velocities were inverted to obtain elastic constants. The authors used a commercial parameter identification software, which was used to perform a least squares iteration followed by a simplex algorithm. Balasubramaniam [9] described the efforts made to reconstruct material stiffness properties of unidirectional fiber-reinforced composites from obliquely incident ultrasonic bulk wave data, employing an inverse technique based on Genetic Algorithm (GA) that does not depend on initial guesses and provides global solutions to complex forward functions [10–14]. The theoretical velocities [10] were calculated using the known elastic constants and these were then used for reconstruction of elastic constants. The authors [10] concluded that elastic constant identification was unconditionally stable up to 2–5% noise of any distribution and noise level of 10% was also acceptable.

Chiroiu et al. [12] explained a non-linear inverse method based on GA to determine second and third order elastic constants of monoclinic materials using ultrasonic velocity data. The authors calculated elastic constants by reducing the error between theoretical and experimental velocities. Silva et al. [13] estimated the elastic constants based on the adjustment of coefficients in an optimization process, i.e. GA in which the objective function is based on the difference between the analytical natural frequencies and the measured ones. The GA based inversion has also been used for the determination of the ply-lay-up in multi-layered composite plates using the dispersion relationship measurements of guided waves [15].

In the present study, unidirectional graphite–epoxy and quasi-isotropic graphite–epoxy $(+45, -45, 0, 90)_{7s}$ composite plates are considered and the ultrasonic phase velocities are measured using back-reflection technique [16]. The inversion is carried out using Genetic Algorithm (GA) which avoids the computation of gradients involving velocity values. The elastic constants obtained are checked for physical applicability by satisfying constraints [17].

2. Theoretical background

The plane wave propagation characteristics and elastic constants are related by the Christoffel equation [3] as

$$k^2 \Gamma_{ij} v_j = \rho \omega^2 \delta_{ij} v_j \tag{1}$$

where k is the wave vector (unit vector normal to the wave front); ω, the angular frequency; v_j, the polarization vector; ρ, the density of material; Γ_{ij}, the Christoffel matrix, given by

$$\Gamma_{ij} = l_{i\mathrm{K}} C_{\mathrm{KL}} l_{\mathrm{L}j} \tag{2}$$

where C_{KL} is the elastic constant matrix; l_{iK}, the direction cosine matrix.

The elements of the Christoffel matrix are functions of the plane wave propagation direction and the elastic constants of the plate. The reciprocal of phase velocity magnitude, as a function of propagation direction is the slowness curve. It is an important tool in ultrasonic non-destructive evaluation since the energy of the wave always travels normal to this slowness curve. The slowness curve can be used to determine the trends in the angular variations of the velocity.

Unlike the anisotropic crystals, composites are made using layers of same material oriented in different directions. Anisotropic crystals can be cut in different directions to propagate wave and measure the velocity. But in general, thin-layered composites cannot be cut in different directions, so the velocities at different propagation directions are obtained by using the oblique incidence immersion setup.

The elastic stiffness matrix for orthorhombic symmetry (orthotropic) materials can be represented in matrix form as

$$[C] = \begin{bmatrix} C_{11} & C_{12} & C_{13} & & & \\ C_{12} & C_{22} & C_{23} & & & \\ C_{13} & C_{23} & C_{33} & & & \\ & & & C_{44} & 0 & 0 \\ & & & 0 & C_{55} & 0 \\ & & & 0 & 0 & C_{66} \end{bmatrix} \tag{3}$$

3. Goniometry based immersion setup

Schematic of the back-reflection technique is shown in Fig. 1. In the Fig. 1, X is the horizontal displacement of the transmitted beam with respect to the incidence ray. At normal incidence, X is zero and increases as the incident angle θ_i increases. θ_r is the wave propagation angle within the plate measured from the normal to the plate.

Back-reflection immersion setup has been used to measure ultrasonic velocity in the material at different angles of propagation. The single axis immersion setup consists of a probe holder, a rectangular frame to hold the sample, a stepper motor and a back-reflector. Probe holder can be adjusted according to the size of the sample. The dimensions of back reflector are selected such that the multiples from back reflector do not interfere with the required signal.

4. Samples description

Well characterized unidirectional graphite–epoxy composite (fibers are aligned along X3-direction) plate of thickness 2.16 mm and 8.4 mm quasi-isotropic graphite–epoxy composite $(+45, -45, 0, 90)_{7s}$ were considered for reconstruction of elastic constants. The 8.4 mm thick plate was made using 56 layers of graphite fiber with the $(+45, -45, 0, 90)$ ply group repeated seven times and then followed by another seven sets of similar ply group represented symmetrically about the middle plane of the laminate. The samples were manufactured under pressure using pre-peg hand lay-up followed by autoclave curing. The two samples were tested for material homogeneity using ultrasonic *C*-scan and were found to be homogeneous and defect free. Ultrasonic velocity measured at a single spot, could be taken as representative of the entire sample, thereby obtaining the effective elastic constants of the medium. Both the samples were modeled as orthotropic materials. This is contrary to the normal assumption that unidirectional fiber composite plates can be modeled using the transversely isotropic model. The justification for this orthotropic model is based on the autoclave nature of the manufacturing process causing the pressure along the normal direction to plate surface to be higher that the tangential direction. A schematic of the quasi-isotropic plate and co-ordinate system used is shown in Fig. 2.

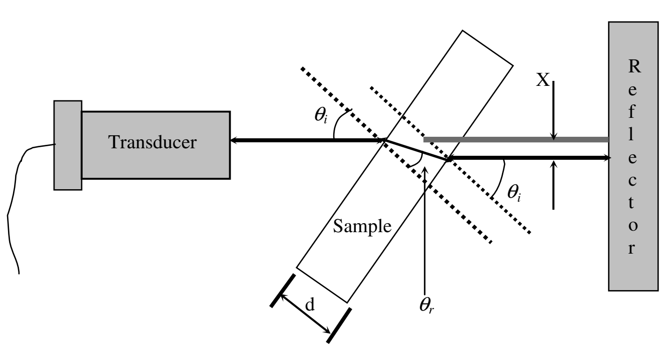

Fig. 1. Schematic of back-reflection technique.

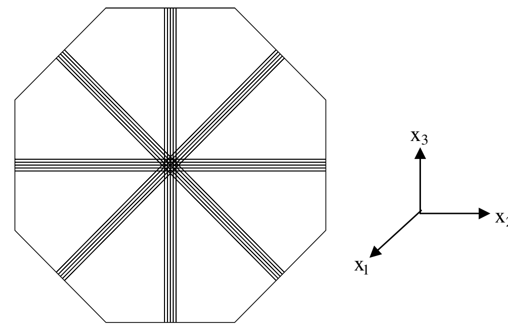

Fig. 2. Fiber orientation and coordinate system for quasi-isotropic graphite–epoxy composite plate.

5. Experimental procedure

A single axis immersion setup has been used to measure the time-of-flight in the sample for different angles of propagation. The ultrasonic velocity at different incident angles was calculated using this time-of-flight data. In the back-reflection technique [16], the same transducer acts as transmitter as well as receiver. Reddy et al. [16] compared the through-transmission and back-reflection ultrasonic immersion techniques for determining the elastic stiffness matrix of materials and concluded that the back-reflection technique is superior to the through-transmission technique. The specimen is held between the transmitter and the back-reflector, and is precisely rotated using a computer controlled stepper motor to give different orientations for the incidence (and propagation) of ultrasonic waves in the plate. The experiments were carried out using a broadband unfocused 10.0 MHz center frequency transducer (V327 Panametrics Inc, USA) for 2.16 mm unidirectional graphite–epoxy composite and a broadband unfocused 1 MHz center frequency transducer (V303 Panametrics Inc, USA) for quasi-isotropic graphite–epoxy composite. Necessary alignments and calibrations were made to the experimental setup to measure ultrasonic time-of-flight data. There were only two modes present in the signal. By time gating the total signal, quasi-longitudinal wave was considered below first critical angle and quasi-shear wave was considered above first critical angle. The reason to select two modes was to reconstruct elastic constants effectively by fitting two slowness curves. The required signal was identified and record length was adjusted such that the reflected signal from the back reflector was isolated and recorded. Specimen was rotated at 1° step resolution, and at each angle of incidence, a total of 256 signals were acquired and averaged (to reduce any transient electronic noise). Further, the effect of "noise" near second critical angle was reduced by using the method of moving window average [18].

For the two samples the ultrasonic time domain (RF) signals were stored at various angles of incidence, from normal incidence to the second critical angle. For each trial, the signal without sample (only through water path) was recorded as the reference signal. The phase velocity of propagation in the plate for the given angles of propagation was calculated using the cross-correlation technique [19,20].

6. Results and discussion

Experiments on samples in three different propagation planes namely 1–2 symmetric plane, 1–3 symmetric plane and a non-symmetric plane at 45° to 1–2, and 1–3 planes (hereby referred to as the 45° plane) were obtained and analyzed. Five trial sets of data were taken for each of the two samples to check for repeatability of the results. Elastic constants reconstructed from these five trial sets shows good agreement with each other and hence only one set of the elastic constants is given. From time-of-flight data, velocities were determined as described in the section below.

6.1. Calculation of velocity and propagation angle

From the measured time-of-flight data, phase velocity in the plate for an incident angle (θ_i) with back-reflection technique is calculated using Eq. (4) [21,22].

$$V_P = \left[\left(\frac{\Delta t}{2d}\right)^2 - \frac{\Delta t/d}{V_w}\cos\theta_i + \left(\frac{1}{V_w}\right)^2\right]^{-1/2} \tag{4}$$

where V_P is the phase velocity (m/s); V_w, the velocity in water (m/s); d the thickness of material (mm); θ_i, the incident angle (in radians); $\Delta t = t_2 - t_1$, the time difference between the signal with (t_2) and without (t_1) sample (s).

The velocity as a function of incident angle in the 1–2 plane of symmetry is shown in Fig. 3. Due to anisotropy of quasi-isotropic plate in 1–2 plane, both the longitudinal and the shear velocities change with incident angle. For the unidirectional plate, the 1–2 plane is perpendicular to the fibers, with both longitudinal and shear velocities reasonably invariant with incident angle. Reddy et al. [16] used this feature in their reconstruction. This approximation has been avoided here and the plate has been modeled as orthotropic to account for the autoclave nature of the manufacturing process as explained in Section 4.

Five sets of phase velocity data in the 45° plane of quasi-isotropic plate was collected using the same experimental arrangement, but at different locations on the sample. All of the data sets are plotted in Fig. 4. It was observed that the variation in these velocity measurements was very small. The standard deviation (σ) of measured velocity is 10.47 m/s for longitudinal wave measured below first

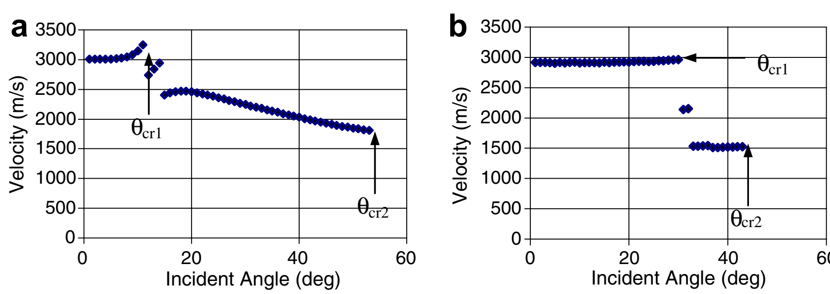

Fig. 3. Velocity vs. incident angle plots for (a) anisotropic 1–2 plane in quasi-isotropic plate and (b) isotropic 1–2 plane in unidirectional plate.

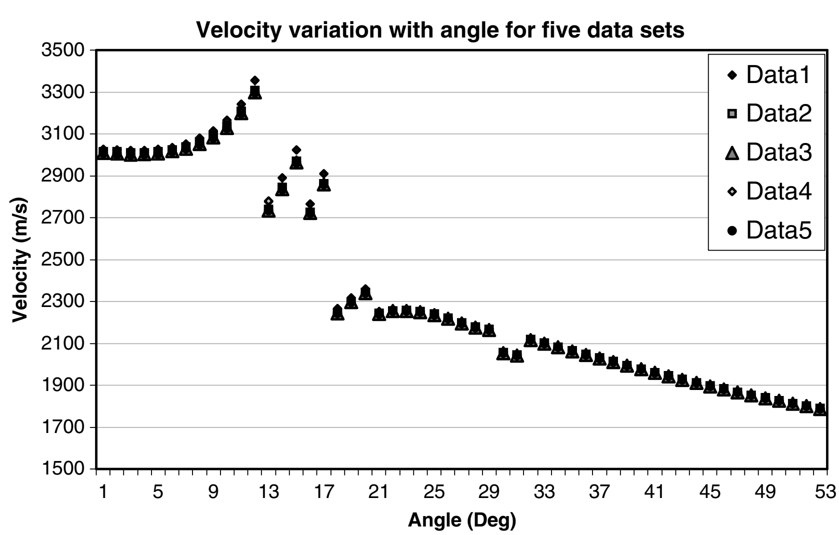

Fig. 4. Five sets of phase velocities in 45° plane of quasi-isotropic plate.

critical angle, and is 1.95 m/s for quasi-shear wave. Similar pattern was observed in unidirectional plate also. The average values of these measurements were used in the elastic constant reconstruction.

The measured phase velocities were used in Snell's law to find the propagation angles (θ_r) in the plate, given by Eq. (5), shows the propagation angle as a function of incidence angle (θ_i).

$$\theta_r = \mathrm{Sin}^{-1}\left(\frac{V_P}{V_w}\mathrm{Sin}\theta_i\right) \tag{5}$$

6.2. Estimation of elastic constants using GA

Propagation angle (θ_r) and propagation velocity (V_P) inside the plate are used in reconstructing the elastic constants of the plate. An Error function [9,10] was first defined as the square of the difference of the squared velocities that were measured and reconstructed (using assumed values of the elastic constants). The Genetic Algorithm (GA) based inverse method of reconstruction then starts with a population of randomly guessed candidate values of the elastic constant set, each of which are then evaluated using the error function. A few of the guesses with the lowest error function are preserved, while the others are placed with new guesses. The subsequent guesses are driven by the Genetic Algorithm evolutionary process described in detail below with the objective of finding the set of elastic constant guesses that provide an error function that is small.

The error function to be minimized, during the inversion of the ultrasonic velocity data, can be represented as follows.

$$\text{Minimize} \quad \mathrm{ERR}(C) = \sum_{i=1}^{i=2}\sum_{j=1}^{j=M}\left[V_{ij}^{c}(\phi,\theta_r)^2 - V_{ij}^{m}(\phi,\theta_r)^2\right]^2 \tag{6}$$

Subject to

$$C_{\mathrm{MIN}} < C < C_{\mathrm{MAX}}$$

where $\mathbf{C}$ is the elastic constant vector; C_{MIN} and C_{MAX} are the lower and upper bounds on the elastic constants; i, an index specifying the mode of bulk wave propagation; j, an index specifying the direction of bulk wave propagation; V_{ij}^{m}, the experimental phase velocity; V_{ij}^{c}, the forward calculated phase velocity; M, the number of different directions of velocity measurements; ϕ, the azimuthal angle with respect to a material symmetry direction and ERR(C), the error function.

Genetic Algorithms have been used to optimize the error function given in Eq. (6). Evolutionary computation techniques such as GA are search algorithms based on the mechanics of natural selection. They are robust, conceptually simple and can be used in situations where mathematical models are unavailable or in situations where the search space is complex to use traditional or conventional techniques. GA is very efficient in finding near global minimum (or maximum).

6.2.1. Creating a sample solution space

In GA, each candidate solution (chromosome) of the solution set (population) undergoes an evolutionary process subject to some fitness criteria. The candidate solutions are coded and supplied to the GA. Each candidate solution (chromosome) consists of 9 unknowns representing the 9 elastic constants. The number of bits per elastic constant is initially chosen as 10 but 20 were used to ensure a good accuracy [10]. The total length of a chromosome is thus 180 bits. At the end of the evolutionary process, the chromosome is decoded for which the 180 bits are divided into 9 substrings of 20 bits each and converted from the binary to a decimal format. Each substring will now represent an elastic constant which is illustrated below for one elastic constant (i.e. C_{11}). Variable ranges are used in decoding a binary code of elastic constants. Variable range for C_{11} is used as 8–16 GPa. Even though a binary coding process was used to create the guessed elastic constants population in the present work, other coding methods such as real coding procedure can also be used.

Binary code of elastic constant, C_{11}																				Decoded value
1	0	1	0	1	1	0	0	1	0	1	1	1	1	0	0	1	1	0	1	13.40

6.2.2. The evolution process

The evolution process can be summarized as the following steps:

1. Evaluate the fitness of each candidate in the current solution set using the error function in Eq. (6).
2. Select two candidate solutions (parents) by using a Roulette wheel selection procedure to combine (reproduce) and produce two new solutions (off springs). Selection of the candidate solutions for reproduction is based on their fitness.
3. Combine the parents to produce two new candidate solutions (the child or offspring) which will combine the characteristics of its parents. The mating process is implemented as a crossover operation whereby bits from the parents' chromosomes are crossed over at random locations. Steps 2 and 3 are repeated until new candidate solution set is filled with children.
4. Some of the newly created candidate solutions are mutated by which bit values of the candidate are randomly flipped from 0 to 1 or vice versa. Such a mutation

operation creates new characteristics in the solution space and prevents the solutions from having characteristics of only the previous generation. In order to ensure the children not to be much different from their parents, only 10 percent mutation is allowed.

5. The principle of Elitism was employed at the time of creating a new generation. Some of the fittest solutions are directly transferred to the next generation without applying crossover and mutation. Elitism carries over good solutions to the next generation. The use of elitism ensures a constant growth of the best fitness in the population, along the generations. In this work, the best chromosome is carried to next generation without modification.

This five step procedure creates the new solution space (next generation).

Sensitivity of the of GA parameters has been checked by executing the code for different sets of GA parameters. The variation of error function $ERR(\boldsymbol{C})$ as a function of the number of generations has been plotted in Fig. 5 for different crossover rates (CR) by keeping mutation rate (MR) as constant at 0.1 and in Fig. 6 for different mutation rates by keeping crossover rate as constant at 0.85. It was observed that the rate of convergence of the error function to a minimum value was highest for GA parameters CR and MR at 0.85 and 0.1 respectively. These optimized GA parameters used in reconstructing the elastic constants are given in Table 1. The GA process was terminated once the number of generations reached 500.

6.3. Reconstruction approach and results

Three different methods for the GA based reconstruction of elastic constants from the measured ultrasonic velocity data were implemented. In all the three methods, measured velocity up to 50° of incidence has been used for the reconstruction.

(a) *Method 1. Orthotropic Assumption and Three Plane Data*: This approach is similar to the one used in [5,7] where velocity is measured at different azimuthal

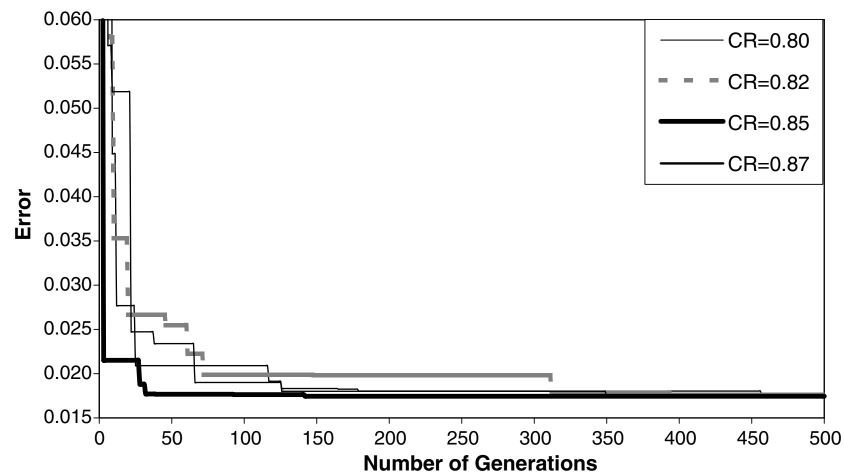

Fig. 5. Variation of error function with number of generations for different crossover rates when mutation rate is fixed at 0.1.

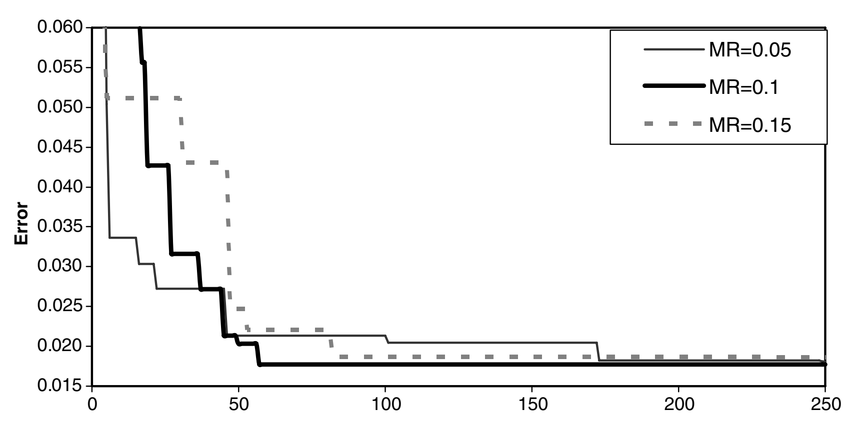

Fig. 6. Variation of error function with number of generations for different mutation rates when crossover rate is fixed at 0.85.

Table 1
Optimized set of GA parameters

GA parameters used in the inversion procedure	
Population size	50
Number of bits per variable	20
Selection type	Roulette wheel
Crossover type	Two point crossover
Crossover rate	0.85
Mutation rate	0.1
Number of elite solutions	1
Number of generations	500

orientations in the 1–2 plane, 1–3 plane and the 45° plane of the sample. The material is assumed to be orthotropic and the reconstruction process is limited to inverting 9 unknown parameters.

(b) *Method 2. Orthotropic Assumption and Single Plane Data*: Velocity in the plate is measured in the 45° non-symmetric plane. The plate is assumed to be orthotropic and the reconstruction process is limited to inverting 9 unknown parameters.

(c) *Method 3. Reconstruction using Triclinic Single Plane method*: Velocity in the plate is measured in the 45° non-symmetric plane. The plate is assumed to be triclinic and the reconstruction process involves inverting 21 unknown parameters.

6.3.1. Method 1 – Orthotropic assumption three plane data

The velocities were measured using back-reflection technique on (a) the 2.16 mm unidirectional graphite–epoxy composite plate and (b) the 8.4 mm quasi-isotropic graphite–epoxy composite plate as representative of orthotropic material. To reconstruct the elastic constants of the orthotropic material using three plane velocity data, the elastic constant C_{11} was calculated using the normal incidence velocity as discussed in Appendix A (Eq. (A3)). The elastic constants C_{22}, C_{12}, and C_{66} were reconstructed by GA using 1–2 plane velocity data, C_{33},C_{13}, and C_{55} were reconstructed by GA using 1–3 plane velocity data. With 7 of the 9 elastic constants determined, the remaining 2 constants C_{44} and C_{23} were reconstructed by GA using 45° non-symmetric plane velocity data.

Slowness curves were generated for wave propagation in 1–2 and 1–3 symmetric planes and 45° non-symmetric plane. The reconstructed elastic constants obtained using 3 plane velocity data and assuming the plate (sample) as orthotropic, were used to draw the slowness as shown in Fig. 7 for the quasi-isotropic plate for propagation in 45° plane. For the two samples, slowness curves generated using forward model are in better agreement with the experimental slowness curves.

Using the current technique of obtaining data from a single plate (irrespective of whether a single plane or multiplane data is used for reconstruction), only seven out of nine elastic constants can be reconstructed. Elastic constants C_{23} and C_{44} cannot be reconstructed as C_{23} and C_{44} are very insensitive to ultrasonic bulk wave velocities in the plate. Even, when C_{23} was varied by 50%, the variation in longitudinal velocity was only 0.15% and the variation in shear velocity was just 0.7%. When C_{44} was varied by 50%, the variation in Longitudinal and shear velocity was only 1%. This has also been reported by others elsewhere [7,10].

6.3.2. Method 2 – Orthotropic assumption and single plane data

Measuring velocity in three planes is difficult since it requires precise rotation of the sample in the azimuthal plane (in addition to the rotation for changing the angle of incidence) and processing multiple sets of data.

To reconstruct the elastic constants of the orthotropic material using single plane velocity data (45° plane), the elastic constant C_{11} is calculated as described in Section 6.3.1. The remaining eight elastic constants are reconstructed by GA using the 45° non-symmetric plane velocity data.

In Fig. 8 slowness curves for 8.4 mm thick quasi-isotropic graphite–epoxy composite for wave propagating in 45° plane were drawn by reconstructing the elastic constants using single plane data assuming orthotropic medium.

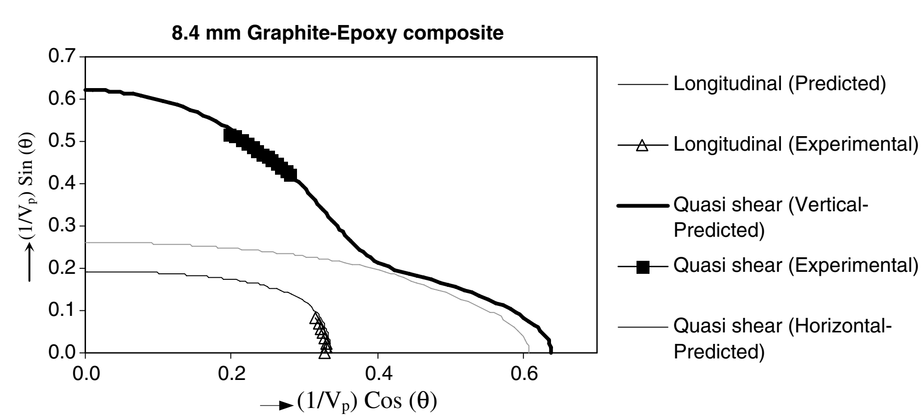

Fig. 7. Slowness curves for 8.4 mm thick quasi-isotropic graphite–epoxy composite for wave propagating in 45° plane using three plane data assuming orthotropic medium.

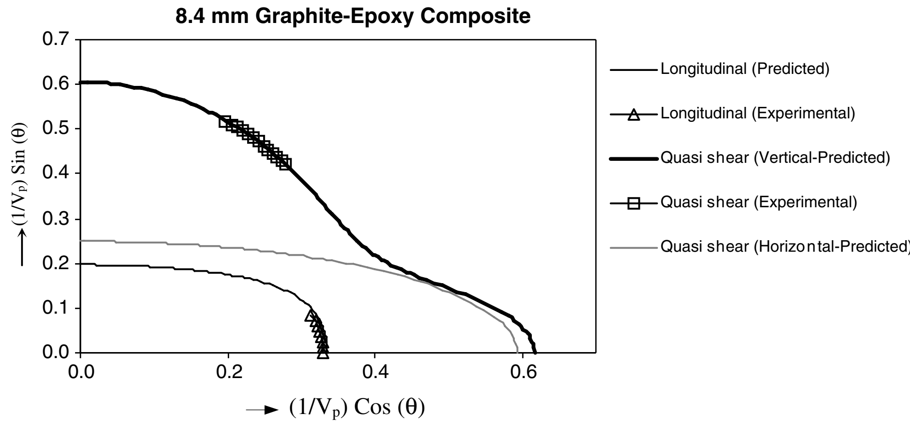

Fig. 8. Slowness curves for 8.4 mm thick quasi-isotropic graphite–epoxy composite for wave propagating in 45° plane using single plane data assuming orthotropic medium.

Checking with slowness curves enables trends to be matched along with the velocity values.

Fig. 8 shows a good agreement between the experimental data and the forward model. Similarity with the slowness curves in Figs. 7 and 8 suggests the possibility of finding elastic constants using single plane data alone.

Knowing the elastic constants of a layer of graphite–epoxy composite material, the theoretical elastic constants of quasi-isotropic graphite–epoxy composite can be calculated by arithmetic averaging [23–25]. The reconstructed elastic constants using three plane data and single plane data are compared with the theoretical elastic constants in Tables 2 and 3 for the quasi-isotropic and unidirectional plates respectively.

The comparison shows the applicability of single plane reconstruction as a quick and reasonable assessment of

Table 2
Reconstructed elastic constants of graphite–epoxy composite plate (8.4 mm thick) using single, and three plane data, compared with the theoretical elastic constants (percentage error from theoretical values in parenthesis) and standard deviation during inversion caused by the stochastic nature of the GA inversion

	Elastic constants (in GPa)						
	Theoretical	Method 1		Method 2		Method 3	
		(% Error)	(σ)	(% Error)	(σ)	(% Error)	(σ)
C_{11}	12.43	13.69 (10.13)	0.00	13.69 (10.13)	0.00	12.71 (2.25)	0.00
C_{12}	5.81	5.98 (2.93)	0.10	6.22 (7.05)	0.10	5.89 (1.37)	0.11
C_{13}	5.81	6.88 (18.42)	0.07	6.75 (16.17)	0.07	5.96 (2.58)	0.13
C_{22}	59.11	60.17 (1.79)	0.92	58.86 (0.42)	0.92	58.07 (1.75)	1.22
C_{23}	20.53	14.47 (29.52)	1.14	10.16 (50.51)	1.14	15.99 (22.11)	0.79
C_{33}	59.11	58.93 (0.30)	1.04	60.59 (2.50)	1.04	57.94 (1.97)	1.37
C_{44}	19.29	4.14 (78.54)	0.41	4.72 (75.53)	0.41	15.77 (18.24)	0.21
C_{55}	4.26	3.80 (10.79)	0.12	4.05 (4.92)	0.12	3.84 (9.85)	0.16
C_{66}	4.26	4.18 (1.87)	0.13	4.36 (2.34)	0.13	3.84 (9.85)	0.13

Table 3
Reconstructed elastic constants of graphite–epoxy unidirectional composite plate (2.16 mm thick) using single, and three plane data, compared with the theoretical elastic constants (percentage error from theoretical values in parenthesis) and standard deviation during inversion caused by the stochastic nature of the GA inversion

	Elastic constants (in GPa)						
	Theoretical	Method 1		Method 2		Method 3	
		(% Error)	STD	(% Error)	STD	(% Error)	STD
C_{11}	12.43	13.48 (8.45)	0.00	14.05 (13.03)	0.00	13.48 (8.45)	0.00
C_{12}	5.39	7.26 (34.69)	0.04	6.64 (23.19)	0.36	5.13 (4.82)	0.21
C_{13}	6.24	6.80 (8.97)	0.02	6.35 (1.76)	0.13	5.34 (14.42)	0.29
C_{22}	12.43	15.39 (23.81)	0.08	14.1 (13.44)	0.57	13.42 (7.96)	0.61
C_{23}	6.24	11.95 (91.50)	0.14	8.29 (32.85)	0.57	8.6 (37.82)	0.42
C_{33}	134.36	135.55 (0.88)	0.36	142.32 (5.92)	1.44	141.0 (4.94)	1.24
C_{44}	5.00	6.43 (28.60)	0.17	8.51 (70.20)	0.46	8.35 (67.0)	0.64
C_{55}	5.00	6.02 (20.40)	0.00	6.82 (36.40)	0.05	6.64 (32.8)	0.08
C_{66}	3.52	3.62 (2.84)	0.03	3.27 (7.10)	0.16	2.65 (24.71)	0.10

elastic constants. The elastic constants reconstructed using GA are given as initial guesses to the Quasi–Newton algorithm (known as BFGS – Broyden, Fletcher, Golfarb, and Shannon algorithm) to minimize an unconstrained multivariable function as mentioned in Eq. (6) without constraints. The gradient search algorithm and Newton's method form the basis of the BFGS algorithm [26]. Quasi–Newton methods begin the search along a gradient line and use gradient information to build a quadratic fit to the model and use a line search method like the Fibonacci method. It was observed that the results from GA are within 0.5% of the results from GA-BFGS. Hence, the results presented in the Table 2 and 3 were considered to be the best solution to the measured data.

6.3.3. Reconstruction using triclinic single plane method

In the Anisotropic Single Plane method, it is essential to obtain the velocity data from a non-symmetry plane such as the 45° plane. Using this single plane velocity data it is possible to reconstruct the different independent elastic constants effectively. The reconstructed elastic constants using this method are shown in Tables 2 and 3. From Tables 2 and 3, it may be observed that the elastic constants could well be reconstructed using data taken from single plane. Assuming orthotropic symmetry plate as triclinic, the feasibility of elastic constants determination is evident from Tables 2 and 3.

Percentage error in the measurement is defined as difference in the theoretical and the reconstructed elastic constants divided by the theoretical elastic constants, and are given in Tables 2 and 3 for quasi-isotropic and unidirectional composite respectively. The error is more for C_{23} and C_{44}, as C_{23} and C_{44} are insensitive to ultrasonic velocity data. Standard deviation (σ) in the reconstructed elastic constants is small for all the constants except C_{23} and C_{44} and is shown in Tables 2 and 3. The variation in the reconstruction is obtained as a result of the stochastic (quasi-random) nature of the GA based inversion, i.e. using the same input velocities, the inversion will not coverage to the same reconstructed elastic constant values. However, the results shown in Table 2 and 3 show that for 7 of the 9 elastic constants the variation is small. It may be noted here that this stochastic behavior is able to provide small variation for the 7 constants that can be reliably measured using the ultrasonic velocities measured here, and may provide a method for the determination of material symmetries without prior knowledge about the material. However, this is beyond the scope of this paper.

Fig. 9 shows the slowness curves drawn using elastic constants reconstructed from single plane data for the quasi-isotropic plate for waves propagating in the 45° plane data assuming anisotropic medium. Similarities in the slowness curves in Figs. 8 and 9 with that in Fig. 7 shows the possibility of finding elastic constants of anisotropic plate effectively using single plane data.

The percent velocity error in the reconstruction was defined as the ratio of the difference between the experimental velocity and the theoretical velocity (calculated using the Christoffel equation with the reconstructed elastic constants) to the theoretical velocity. This percent velocity error was used as a measure of the quality of the reconstruction of the elastic constants from the measured ultrasonic velocity data. The percent velocity error using the above three methods is tabulated in Table 4.

The reconstructed elastic constants were then verified based on the engineering constants data provided for the unidirectional laminates by the manufacturer of the composite materials that were obtained using a statistical

Table 4
Percentage velocity error (as defined in Section 6.3.3) in longitudinal and shear velocities

Method	% Errors in	
	Longitudinal velocity	Shear velocity
Method 1	0.51	0.16
Method 2	0.4	0.1
Method 3	1.7	0.06

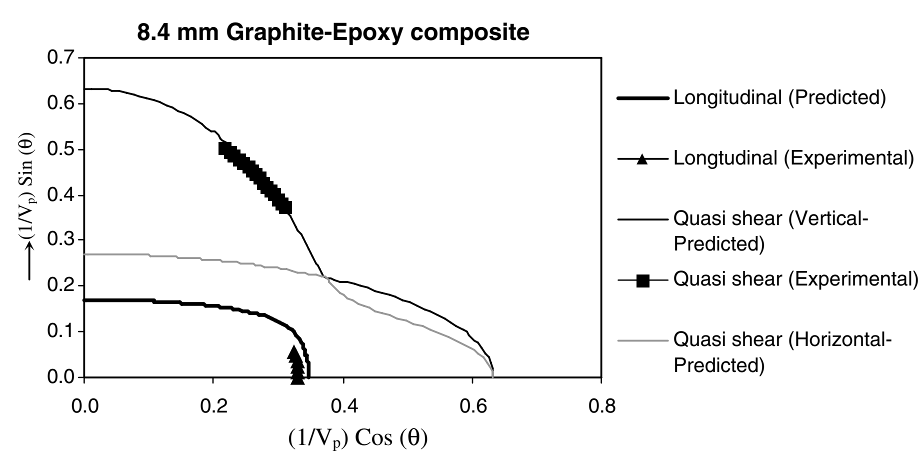

Fig. 9. Slowness curves for 8.4 mm thick composite for wave propagating in 45° plane using anisotropic single plane method assuming anisotropic medium.

Table 5
Percentage error in reconstructed engineering constants of unidirectional graphite–epoxy composite

	Engineering constants (unidirectional composite)						
	Manufacturer data	Method 1		Method 2		Method 3	
		Reconstructed	Error	Reconstructed	Error	Reconstructed	Error
E_{11}	10	10.04	0.45	10.88	8.80	11.49	14.88
E_{33}	130	126.14	2.97	136.90	5.31	135.12	3.94
G_{13}	5	6.02	20.40	6.82	36.40	6.64	32.80

Table 6
Percentage error in reconstructed engineering constants of quasi-isotropic graphite–epoxy composite

	Engineering constants (quasi-isotropic)						
	Inferred from manufacturer data	Method 1		Method 2		Method 3	
		Reconstructed	Error	Reconstructed	Error	Reconstructed	Error
E_{11}	11.58	12.56	8.46	12.49	7.79	11.76	1.54
E_{33}	50.77	53.19	4.77	56.36	11.02	51.98	2.40
G_{13}	4.26	3.80	10.80	4.05	4.93	3.84	9.86

number of tension tests. Since the stiffness constants (C_{ij}) data were not available from the manufacturer, and only three constants for the unidirectional materials were provided, the reconstructed stiffness constants were converted into the engineering constants and compared with the manufacturer data in Table 5 for the unidirectional composite sample. It can be noted that the E_{11} and the E_{33} compare well, but the G_{13} shows an error of approximately 20–35%. It was however further clarified by the manufacturer that only the E_{11} and E_{33} were measured using the tension test and the G_{13} constant was estimated based on experience.

The manufacturer data was not available for the quasi-isotropic sample. Hence, the simple "rule of mixtures" was used by coordinate transformation of the unidirectional elastic constants and by averaging them based on the ply lay-up. Then this "inferred" data was compared with the engineering constants that were obtained from the reconstructed stiffness constants using the 3 methods and is shown in Table 6. Again, from Table 6, it was observed that the E_{11} and the E_{33} were reconstructed well, while the G_{13} showed higher % error.

7. Summary

A GA based inversion of ultrasonic velocity measurements obtained from laminated fiber reinforced composite plates using a oblique incidence back-reflection technique to reconstruct elastic constants has been demonstrated. Two samples, a (i) 8.4 mm thick quasi-isotropic graphite–epoxy composite and a (ii) 2.16 mm unidirectional graphite–epoxy composite plate were made. Velocities were measured as a function of angle and three different methods were described in detail to reconstruct the elastic constants. Derivations of the forward model to calculate velocities from elastic constants and the inverse model to obtain elastic constants from velocity data are also provided.

The Method for calculating elastic constants using three plane velocity data and single plane velocity data while assuming the medium as orthotropic showed that the single plane reconstruction in a non-symmetric plane was sufficient to calculate all seven elastic constants of an orthotropic medium with an accuracy comparable with a more time consuming procedure using data sets from multiple planes (since the sample or the apparatus must be rotated).

This work may be further extended to complex anisotropy cases like a GA based reconstruction of the more general case involving 21 elastic constants (corresponding to triclinic symmetry). Such an effort may require more precise measurement of the velocities and use of very broadband transducers. Velocity measurements can be made automated and the reconstruction procedure cost effective. Such efforts have already been made [27] using PVDF based transducers. The inversion procedure can also be extended to other symmetries, like the Monoclinic and Triclinic symmetries.

Acknowledgements

The authors thank the Advanced Composite Division of the National Aeronautical Laboratory, Bangalore for providing the composite samples, and Department of Science and Technology (DST), New Delhi for funding this research work.

Appendix A

A.1. Orthotropic material

For a wave propagating in 45° plane, direction cosines are givenby

$$l_1 = \mathrm{Cos}\theta,\ l_2 = \mathrm{Sin}\theta\,\mathrm{Cos}\phi\ l_3 = \mathrm{Sin}\theta\,\mathrm{Sin}\phi,\ (\text{Here } \phi = 45^\circ) \qquad (A1)$$

Here ϕ is the angle made by incident ray with X_2-axis and θ is the angle made by incident ray with X_1-axis.

From Christoffel equation, $|\Gamma_{ij} - \rho v^2 \delta_{ij}| = 0$,

$$\begin{vmatrix} a_1 - \rho v^2 & b_1 & c_1 \\ a_2 & b_2 - \rho v^2 & c_2 \\ a_3 & b_3 & c_3 - \rho v^2 \end{vmatrix} = 0$$

where

$$a_1 = C_{11} l_1^2 + C_{55} l_3^2 + C_{66} l_2^2$$
$$b_1 = (C_{12} + C_{66}) l_1 l_2$$
$$c_1 = (C_{13} + C_{55}) l_1 l_3$$
$$a_2 = b_1$$
$$b_2 = C_{22} l_2^2 + C_{44} l_3^2 + C_{66} l_1^2$$
$$c_2 = (C_{23} + C_{44}) l_2 l_3$$
$$a_3 = c_1$$
$$b_3 = c_2$$
$$c_3 = C_{33} l_3^2 + C_{44} l_2^2 + C_{55} l_1^2$$

After simplification, the equation reduces to

$$x^3 + A_2 x^2 + A_1 x + A_0 = 0 \quad \text{(A2)}$$

where

$$x = \rho v^2$$
$$A_2 = a_1 + b_2 + c_3$$
$$A_1 = -(-a_1 b_2 - a_1 c_3 + c_2 b_3 + a_2 b_1 + a_3 c_1 - b_2 c_3)$$
$$A_0 = -(a_1 b_2 c_3 - a_1 c_2 b_3 - a_2 b_1 c_3 + a_2 c_1 b_3 + a_3 b_1 c_2 - a_3 c_1 b_2)$$

By solving the cubic equation, three roots corresponding to one quasi longitudinal and two quasi shear velocities can be obtained.

By substituting $\phi = 90$ and $\phi = 0$ in Eq. (A1), and simplifying the Christoffel equation, gives the velocity expressions in 1–3 plane and 1–2 plane as given in Eqs. (A4) and (A5) respectively. These expressions are explicitly provided in Auld [3].

A.2. Inverse model

Elastic constants were reconstructed from the experimentally measured ultrasonic velocity data. Inversion procedure for elastic constants of orthotropic material involves the determination of nine independent elastic constants. Nine elastic constants were determined in four steps.

Step 1: C_{11} is determined from normal incidence velocity using the following relation

$$C_{11} = \rho v_L^2(0) \quad \text{(A3)}$$

Step 2: Inverting both longitudinal and shear velocity data measured in 1–3 plane using GA, by minimizing the norm of error between theoretical and experimental squares of velocities, it is possible to determine C_{13}, C_{33}, C_{55}. Experimental velocity data is fitted in the following two relations to determine these three unknowns.

$$v_L = \sqrt{\frac{B + \sqrt{B^2 - 4c}}{2\rho}} \quad \text{(A4.a)}$$

$$v_{QS} = \sqrt{\frac{B - \sqrt{B^2 - 4c}}{2\rho}} \quad \text{(A4.b)}$$

where

$$b = -(C_{11} \cos^2\theta + C_{33} \sin^2\theta + C_{55}) = -B$$
$$c = C_{11} C_{55} \cos^4\theta + C_{33} C_{55} \sin^4\theta + \frac{\sin^2 2\theta}{4}[C_{11} C_{33} + C_{55}^2 - (C_{13} + C_{55})^2]$$

Step 3: As in step 2, inverting both longitudinal and shear velocity data measured in 1–2 plane using GA, by minimizing the norm of error between theoretical and experimental squares of velocities, it is possible to determine C_{12},C_{22},C_{66}. Experimental velocity data is fitted in the following two relations to determine the three unknown parameters.

$$v_L = \sqrt{\frac{B + \sqrt{B^2 - 4c}}{2\rho}} \quad \text{(A5.a)}$$

$$v_{QS} = \sqrt{\frac{B - \sqrt{B^2 - 4c}}{2\rho}} \quad \text{(A5.b)}$$

where

$$b = -(C_{11} \cos^2\theta + C_{22} \sin^2\theta + C_{66}) = -B$$
$$c = C_{11} C_{66} \cos^4\theta + C_{22} C_{66} \sin^4\theta + \frac{\sin^2 2\theta}{4}[C_{11} C_{22} + C_{66}^2 - (C_{12} + C_{66})^2]$$

Step 4: By inverting both longitudinal and shear velocity data measured in the 45°-plane using GA, the elastic constants C_{23} and C_{44} are determined. Experimental velocity data is fitted in to the highest and lowest velocities obtained from the Eq. (A2).

All of the elastic constants of orthotropic plate can be reconstructed by inverting the velocities in single non-symmetric plane such as 45° plane using the velocity expressions given in Eq. (A2).

References

[1] Reddy JN. Mechanics of laminated composite plates: theory and analysis. USA: CRC Press; 1991.

[2] Rose JL. Ultrasonic waves in solid media. United Kingdom: Cambridge University Press; 1999.

[3] Auld BA. Acoustic fields and waves in solids. Malabar, Florida, USA: Robert E. Krieger Publishing Company; 1990.

[4] Kline RA. Nondestructive characterization of composite media, Lancaster. Pennsylvania, USA: Technomic Publishing Company Inc; 1992.

[5] Chu YC, Rokhlin SI. Stability of determination of composite moduli from velocity data in planes of symmetry for weak and strong anisotropies. J Acoust Soc Am 1994;95(1):213–25.

[6] Ditri JJ. On the determination of the elastic moduli of anisotropic media from limited acoustical data. J Acoust Soc Am 1994;95(4):1761–7.

[7] Chu YC, Degtyar AD, Rokhlin SI. On determination of orthotropic material moduli from ultrasonic velocity data in non-symmetry planes. J Acoust Soc Am 1994;95(6):3191–202.
[8] Balasubramaniam K, Whitney SC. Ultrasonic through- transmission characterization of thick fiber-reinforced composites. J NDT E Int 1996;29(4):225–36.
[9] Balasubramaniam K. Experimental determination of material stiffness constants for advanced composites using genetic algorithms. Proc SPIE Int Soc Opt Eng 1997;2921:116–9.
[10] Balasubramaniam K, Rao NS. Inversion of composite material elastic constants from ultrasonic bulk wave phase velocity data using genetic algorithms. Compos Part B 1998;29B(2):171–80.
[11] Goldberg DE. Genetic algorithms in search, optimization, and machine learning. Reading, MA: Addison-Wesley; 1989.
[12] Chiroiu C, Munteanu L, Chiroiu V, Delsanto PP, Scalerandi M. A genetic algorithm for determination of the elastic constants of a monoclinic crystal. Inverse Problems 2000;16:121–32.
[13] Silva MFT, Borges LMSA, Rochinha FA, De Carvalho LAV. A genetic algorithm applied to composite elastic parameters identification. Inverse Probl Sci Eng 2004;12(1):17–28.
[14] Liu GR, Ma WB, Han X. An inverse procedure for determination of material constants of composite laminates using elastic waves. Comput Meth Appl Mech Eng 2002;191(33):3543–54.
[15] Balasubramaniam K. Inversion of ply lay-up sequence for multi-layered fiber reinforced composite plates using Genetic Algorithm. Nondestr Test Eval 1999;15:311–31.
[16] Reddy SSS, Balasubramaniam K, Krishnamurthy CV, Shankar M. Ultrasonic goniometry immersion techniques for the measurement of elastic moduli. J Compos Struct 2005;67:3–17.
[17] Jones RM. Mechanics of composite materials. New York: Hemisphere Publishing Corporation; 1975.
[18] Bracewell R. The Fourier transform and its applications. NY: McGraw-Hill; 1969.
[19] Rao BPC, Jayakumar T, Bhattacharya DK, Raj B. New methodology for precise ultrasonic velocity measurement and its applications. J Pure Appl Ultrasonics 1993;15:53–9.
[20] Castagnede B, Roux J, Hosten B. Correlation method for normal mode tracking in anisotropic media using an ultrasonic immersion system. Ultrasonics 1989;27:280–7.
[21] Slotwinski JA, Blessing GV. Ultrasonic measurement of the dynamic elastic moduli of small metal samples. ASTM J Test Eval 1999;27(2):164–6.
[22] Papadakis EP. Ultrasonic wave measurements of Elastic moduli E, G, and μ for Product Development and Design Calculations. J Test Mater 1998;26(3):240–6.
[23] Lhermitte TD, Perrin B. Anisotropy of the elastic properties of cross-ply fiber-reinforced composite materials. IEEE Ultrason Symp Proc 1991;2:825–30.
[24] Tsai SW. Composite Design, Think Composites, Paris, 1985.
[25] Liu GR, Xi ZC. Elastic waves in anisotropic laminates. USA: CRC Press; 2002.
[26] Ralph WP. Optimization for Engineering Systems. Available from: <http://www.mpri.lsu.edu/bookindex.htm>.
[27] Kumar PP, Krishnamurthy CV, Balasubramaniam K. Elastic constants inversion from Ultrasonic transmission spectra obtained using a wide aperture PVDF sensor. Review of Progress in Quantitative Nondestructive Evaluation, AIP Proceedings, 2006; 25(A): 1127–32.