SIMULATION OF ULTRASONIC TECHNIQUE USING SPECTRAL ELEMENT METHOD

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ABSTRACT: Numerical simulation of ultrasonic wave propagation using methods such as finite element or finite difference is computationally expensive particularly when (a) structural dimensions are high, (b) inspection at higher frequencies (due to short wavelengths), and (c) in complex materials that are not isotropic. This paper discusses a numerical technique, which is similar to FEM, but works in frequency domain and has advantage of more accurate results in quick computational time called the spectral element method (SEM). When the second order partial differential wave equation transformed to frequency domain by Continuous Fourier Transform, the wave equation transforms to ordinary differential equation (ODE) that has exact solution. This paper discusses simulation of Lamb wave modes and Time of Flight Diffraction Technique in isotropic plates.

Keywords: FEM, ultrasonics, spectral element
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INTRODUCTION

For an applied force with a high-frequency content to simulate higher frequencies (>1 MHz), the associated wavelengths become small at high frequencies. In order to capture all the higher modes, the usual finite element model (FEM) requires very small element size to match the wavelength. This increases the system size enormously, and hence the conventional finite element method becomes very difficult computationally. In addition, this discretized model is exposed to crude error-bound approximation due to the numerical stability limit in computation [1,2]. Numerical simulation of wave propagation and their interaction with defects in the following items are very difficult using regular finite element method.

a) Large structure e.g. wave propagation in rails
b) Structure other than isotropic e.g. composite in a SHM system
c) High frequency inspection e.g. TOFD inspection of thin sections

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To solve these kinds of problems efficiently without loss of accuracy, a model based on frequency domain finite element methodology is developed called the Spectral Element Method (SEM).

SEM is similar to FEM, but works in frequency domain and has the advantage of giving more accurate results in quick computational time. Conventional finite elements treat the dynamic load induced by the mass and rotational inertia of the beam as concentrated loads and moments applied at the ends of the element. Even though the structural joints may be far apart, many elements must be used if the inertia distributions are to be modeled accurately. Therefore, the number of elements required to do a dynamic problem is substantially larger than that required for the equivalent static one [3]. The spectral analysis formulates an element, which treats the distribution of mass and rotational inertia exactly. Only one spectral element need to be placed between any two joints, substantially reducing the total number of degrees of freedom in the system.

SEM formulation uses the fast Fourier transform (FFT) algorithm to transform the distributed parameters from time domain to frequency domain and vice versa [1]. When the second order partial differential wave equation transform to frequency domain by Continuous Fourier Transform, the wave equation transforms to ordinary differential equation (ODE) that has exact solution.

In SEM, the structure is divided into number of waveguides with connectivity at the joints. The energy in a waveguide is directed along its length. The waveguide that has one degree of freedom (rod element) simulates the longitudinal motion. The waveguide with two degrees of freedom (modified rod element) simulates the shear wave and a waveguide with three degrees of freedom (beam element) simulates the flexural motion [1]. The beam wavegudes were developed using Euler- Bernoulli theory [1,2] and Timoshenko beam theory [4] for different applications.

This Paper discusses application of spectral element method to simulate ultrasonic wave propagation [1,2 and 3]. Normal and angle beam techniques are used to simulate guided wave technique.

FORMULATION

Simulation of normal and angle beam ultrasonic technique using SEM is similar to 2-D FEM other than

1. Instead of dividing the entire structure into number of elements, the structure is divided into number of waveguides
2. The force applied at different nodes with in FEM is applied as equivalent forces at the nodes of the beams in SEM.

When dividing the structure into appropriate waveguides, the stiffness matrix for different waveguides (linear and tapered) is derived in the frequency domain. The stiffness matrixes for linear and tapered beam elements are given in [1-3].

The 1-D wave equation in an isotropic solid is governed by a partial differential equation (PDE) given by,

$$\frac{\partial^2 u(x,t)}{\partial x^2} = a^2 \frac{\partial^2 u(x,t)}{\partial t^2} \tag{1}$$

where

$$a = \sqrt{\frac{E}{\rho}}$$

Applying a continuous fourier transform to equation 1 gives

$$\frac{d^2\hat{u}(x,\omega)}{dx^2} + \omega^2 a^2 \hat{u}(x,\omega) = 0 \qquad (2)$$

Equation 2 is an ordinary differential equation (ODE) with constant coefficients. When the PDE is transformed to ODE, it can be solved easily. The Equation 2 has the general solution of the form,

$$\hat{u}(x,\omega) = C_1 e^{i\omega ax} + C_2 e^{-i\omega ax} \qquad (3)$$

An inverse transform of the Equation 3, would give as the original time domain signal as shown below,

$$u(x,t) = \int_{-\infty}^{\infty} \{C_1 e^{i\omega ax} + C_2 e^{-i\omega ax}\} e^{i\omega t} d\omega$$

$$= \int_{-\infty}^{\infty} C_1 e^{i\omega(ax+t)} d\omega + \int_{-\infty}^{\infty} C_2 e^{-i\omega(ax-t)} d\omega \qquad (4)$$

where C_1 and C_2 are constants can be obtained from boundary conditions.

In SEM, $\hat{u}(x,\omega)$ is interpolated between nodes and a spectral stiffness matrix is derived with respect to $\hat{u}(x,\omega)$. $\hat{u}(x,\omega)$ is solved using matrix equation as explained below

Consider element of length ‘L’ with two nodes as shown in Figure 1 and the displacement end conditions are

$$\hat{u}(0,\omega) = \hat{u}_1 = C_1 + C_2 e^{-ikL} \qquad (5)$$

$$\hat{u}(L,\omega) = \hat{u}_2 = C_1 e^{-ikL} + C_2 \qquad (6)$$

$\hat{u}_1$ and $\hat{u}_2$ are the nodal displacements or degrees of freedom and $k = \omega a$ is the wave mode or wave number.

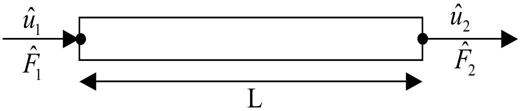

FIGURE 1. Nodal loads and degrees of freedom for rod element.

The Equation 5 and 6 can be modified and written in the matrix form as

$$\begin{Bmatrix} C_1 \\ C_2 \end{Bmatrix} = \frac{1}{(1-e^{-i2kL})}\begin{bmatrix} 1 & -e^{-ikL} \\ -e^{-ikL} & 1 \end{bmatrix}\begin{Bmatrix} \hat{u}_1 \\ \hat{u}_2 \end{Bmatrix} \tag{7}$$

The displacements and member forces are obtained by simply using the relation

$$\hat{F}(L,\omega) = EA\frac{d\hat{u}}{dx} \tag{8}$$

By taking inverse of Equation 7 and substituting in Equation 8, the matrix form of dynamic stiffness of the two-noded element can be written as

$$\begin{Bmatrix} \hat{F}_1 \\ \hat{F}_2 \end{Bmatrix} = \frac{EAik}{(1-e^{-i2kL})}\begin{bmatrix} 1+e^{-i2kL} & -2e^{-ikL} \\ -2e^{-ikL} & 1+e^{-i2kL} \end{bmatrix}\begin{Bmatrix} \hat{u}_1 \\ \hat{u}_2 \end{Bmatrix} \tag{9}$$

This can be written in the familiar form

$$\hat{\boldsymbol{F}} = \hat{\boldsymbol{k}}\hat{\boldsymbol{u}} \tag{10}$$

where $\hat{\boldsymbol{k}}$ is the frequency dependent dynamic element stiffness. The applied force is also transformed to frequency domain and applied over the frequency of interest. The resultant displacement in frequency domain is then transformed to time domain by FFT algorithm. The phase shift in the frequency domain due to movement in the time domain is monitored.

SIMULATION OF DEFECTS

Structure with linear defects (delamination in composites) can be divided into four linear beams as shown in Figure 2 [5]. The interface compatibility condition is arrived at by assuming that the cross section remains constant and straight. By assuming this condition, the interface nodal displacements are written in terms of other nodal values (thickness times rotation). The structure with horizontal defects is a special configuration of an inclined crack, which is being developed in the next section.

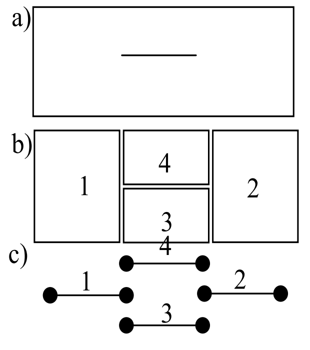

FIGURE 2. SEM solution of structure with horizontal defect. a) structure with horizontal defect, b) waveguide solution and c) idealization.

The simulated signals for a horizontal defect of lengths a) 10 mm, b) 15 mm, c) 20 mm, d) 25 mm, e) 30 mm, f) 35 mm, g) 40 mm and i) 45 mm in a 20 mm thickness plate of length 250 mm is shown in Figure 3. In the figure 1, 2, 3 and 4 are lateral wave, signal from left tip of the defect, signal from right tip of the defect and reflected backwall echo respectively. A simple case of horizontal defect in the same configuration was simulated using FEM and important observation was, the computational time using FEM requires 90 minutes compared to 2 minutes using SEM (97% reduction in computational time). The comparison of results is shown in Figure 4 for 10 mm thick aluminium sample with 5 mm length defect.

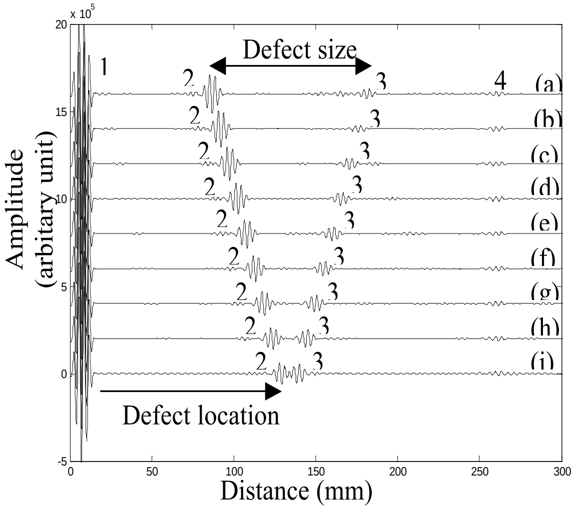

FIGURE 3. Simulated signals using SEM for different length of horizontal defect in a flat aluminium plate of thickness 20 mm.

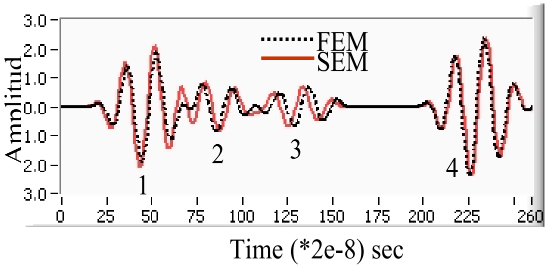

FIGURE 4. Comparison of 2-D FEM with 1-D SEM for a 5 mm horizontal defect in a 10 mm thick aluminium plate.

A two nodded spectral beam element is used to simulate lamb wave modes. Figure 5 compares the simulated results with experiments. Angle beam is simulated to generate particular mode and the simulated results are compared with experiments in figure 6.

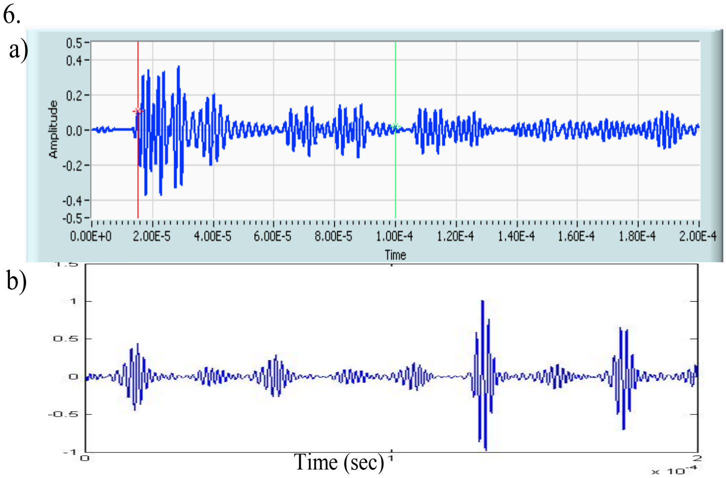

FIGURE 5. Comparison of a) experimental results with b) simulated. Lamb Waves on 1 mm thick Aluminum plate using 500 kHz normal beam transducer.

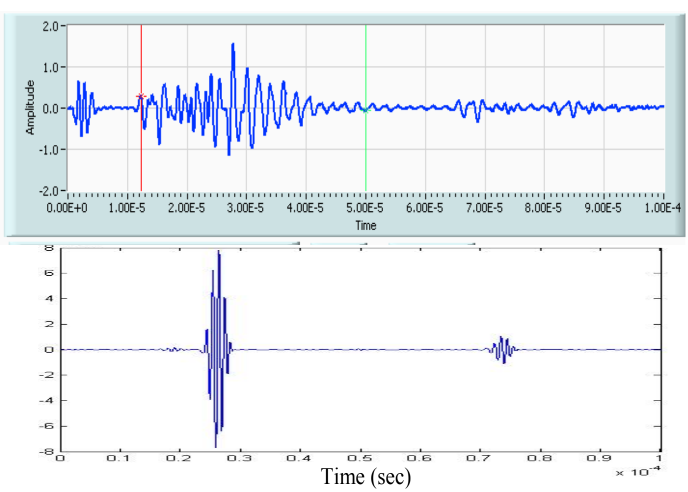

FIGURE 6. Comparison of a) experimental results with b) simulated. Lamb Waves on 1 mm thick Aluminum plate using 500 kHz normal beam transducer.

SUMMARY

The applications of SEM to simulate ultrasonic signatures were demonstrated. The important advantage of SEM is in simulation in quick time. Preliminary work has been done to simulate Lamb wave modes using SEM to highlight the ability of SEM to capture different modes. Normal and angle beams were simulated using equivalent nodal weights. The simulated results were compared with FEM and shows considerable decrease in computational time.

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